Backwards is better: the circumbinary stable zone is larger for retrograde orbits

Simon Edgeworth
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A circumbinary planet is a planet which orbits around *both* stars of a binary star system.

The first discovery of a circumbinary planet was in 1993 [1]. Since then the number discovered has risen to over a dozen [2]. And it is anticipated that the European Space Agency's Gaia mission may discover many more [3].

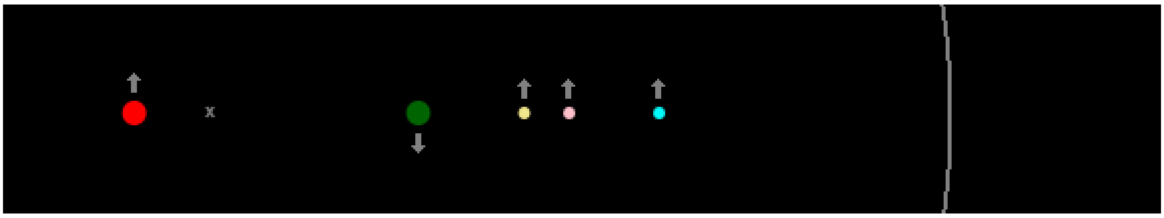
If a circumbinary planet's orbit is large relative to the distance between the two stars, orbit stability fairly simple. It is much like orbiting a single star, with the addition of a slight wobble in the orbit, and the orbit is still stable.

However if the planet's orbit is relatively small, very close to the two stars, there are large perturbations caused by the complicated rotating gravitational field of the two stars. So the question is: How close to the two stars can a circumbinary planet exist with a stable orbit?

For this investigation, a test binary star system is constructed, comprising the two stars of the real Nu Octantis binary star system [4], and three hypothetical circumbinary planets.

The starting positions for the simulation are shown in Figure 1. The centre of mass of the system is shown as a small cross (grey). The primary star (red) has mass equal to to 1.4 times the mass of our Sun, and the secondary star (green) has mass equal to to 0.5 times the mass of our Sun. The two stars are started at apoapsis (maximum seperation). Orbiting around both the stars are the three hypothetical planets (yellow, pink, blue). The line on the right (light grey) is the inner boundary of the stable zone for prograde orbits [5].

Figure 1: Starting positions and directions of the stars and hypothetical planets



The three hypothetical planets are given retrograde orbits, and are ambitiously positioned well inside the zone where prograde orbits are unstable. (Prograde orbits at these small distances would be immediately unstable).

The three hypothetical planets are assumed to have negligible (zero) mass. This enables the three planetary orbits to be integrated simultaneously, while ensuring that the result for each planet is independent of the presence and behaviour of the other two planets.

For each planet, many initial velocities are tested by integration for a few hundred years. Initial velocities which give unstable orbits are rejected, and initial velocities which give stable orbits are noted. In this way, for each planet the range of initial velocities which give stable orbits is identified. Next, one of those velocities (approximately in the middle of the identified range) is selected for testing over a much longer period. The initial parameters of the simulation are listed in figure 2. The units used are defined in [6].

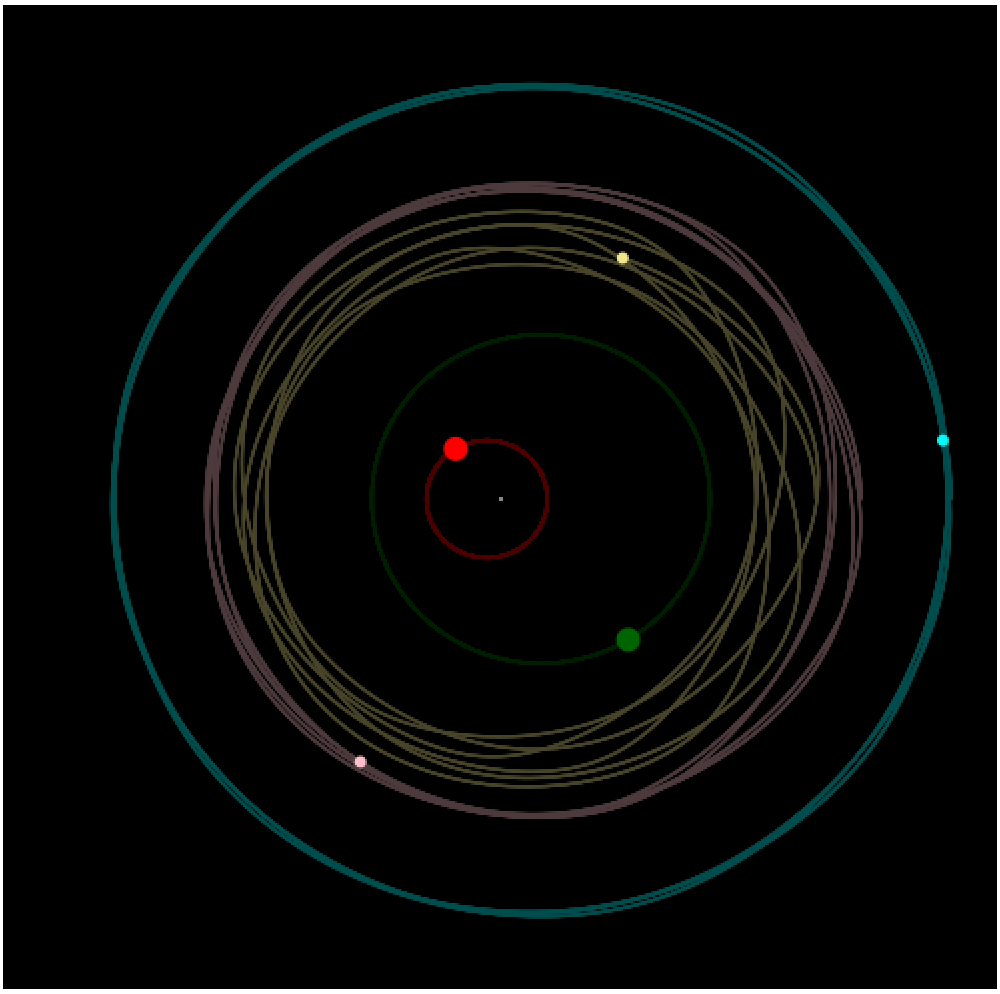
Figure 2: Starting parameters for the simulation

	pX	*vY*	*Mass*
Primary star	-0.829286842105	0.178629233587	1.4
Secondary star	2.32200315789	-0.500161854036	0.5
Hypothetical planet 1	3.5	0.836	0
Hypothetical planet 2	4.0	0.740	0
Hypothetical planet 3	5.0	0.632	0

The situation just after starting the simulation, at about 21 earthyears, is shown in figure 3. The primary star is red, the secondary star is green, and the tiny white dot is the system barycentre. The inner planet (yellow) has completed about 6.2 orbits, the middle planet (pink) has completed about 4.65 orbits, and the outer planet (blue) has completed about 2 orbits.

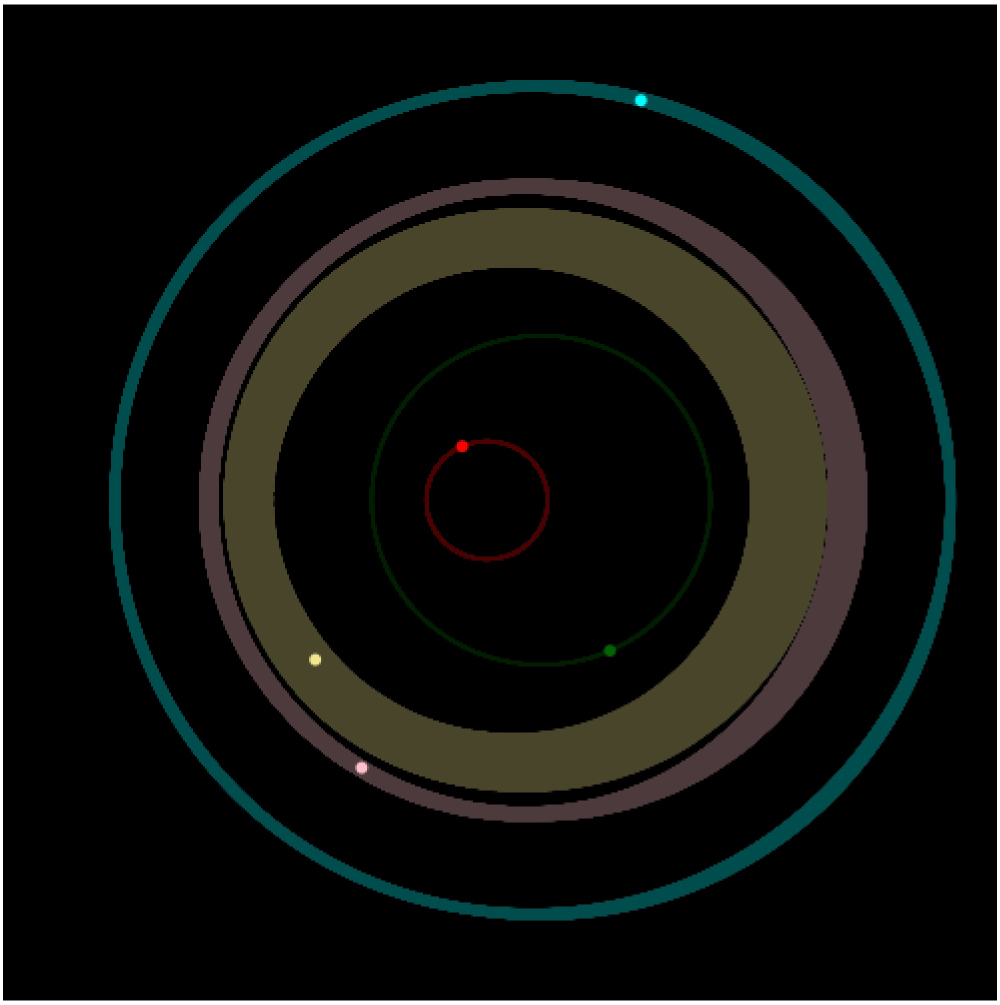
The paths of all three planets have significant deviations from circular paths. Exactly circular orbits are impossible at these distances. Each of the selected orbits has only a small amount of free eccentricity, and the majority of their eccentricity consists of the forced eccentricity which is caused by the complicated rotating gravitational field of the two stars. In other words, these orbits are about as close to circular as is possible for orbits at these small distances from the two stars..

Figure 3: The orbits after about 21 years



The three selected orbits are numerically integrated for 190000 years and the result is shown in in figure 4. The traced path of each planet over time fills an orbital band. The width of the orbital band is greatest for the planet closest to the two stars, and least for the planet furthest from the two stars. Each planet stays always within its orbital band.

Figure 4: The orbital bands after about 190000 years



The orbital bands are not centred on the system barycentre, but are offset in the direction of the apoapsis of the secondary star. This is very similar to the orbital band offset of *circumstellar* orbits within binary star systems (orbits around just one the stars) [7].

The initial velocity of any of the hypothetical planets may be very slightly varied, strictly within a small defined range, and this produces an amended orbit which has slightly different width and shape of its orbital band, and slightly different orbital period, but which is also stable.

The shapes of the orbital bands remain always strictly aligned with the line of apses of the binary star system. If the stellar orbits have apsidal precession (due to the presence of some third massive body) then the shapes of the orbital bands precess at exactly the same rate, and maintain their strict relative orientation to the line of apses of the stellar orbits.

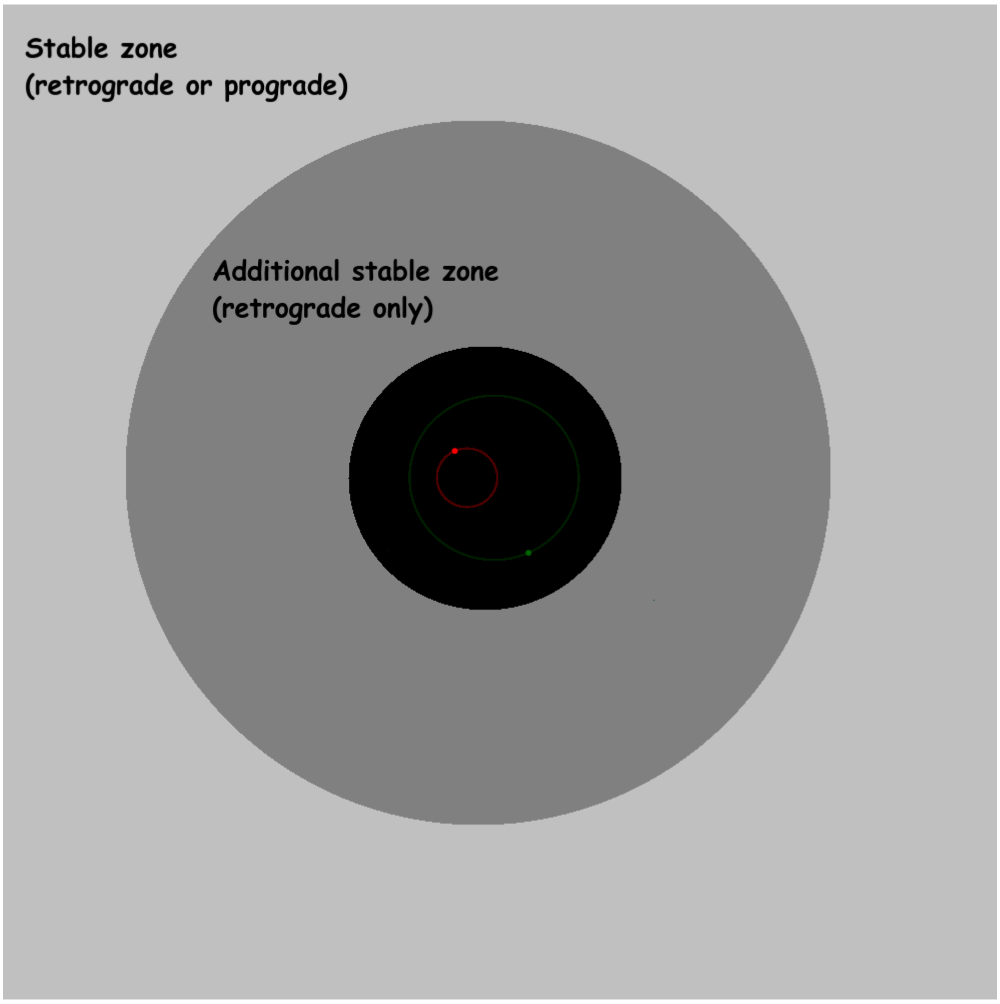
Figure 5 lists the sizes and orbital periods of the orbits of the hypothetical planets, and compares them with the size and period of the stellar orbit, and with the size and period of a prograde orbit at the inner boundary of the prograde stable zone.

Figure 5: Orbit sizes and orbital periods

orbit	*semimajor axis (AU)*	*semimajor axis (ratio to stellar s.m.a.)*	*orbital period (earthyears)*	*orbital period (ratio to stellar orbital period)*
Stellar	2.55	1	2.9	1
Hypothetical planet 1	3.02	1.18	3.42	1.18
Hypothetical planet 2	3.53	1.38	4.57	1.58
Hypothetical planet 3	4.64	1.82	7.03	2.42
Start of prograde stable zone	8.1	3.2	15.6	5.4

The inner boundary of the stable zone for retrograde orbits is then conservatively estimated as being equal to the mean semimajor axis of the orbital band of the inner planet. The resulting enlargement of the stable zone is illustrated in figure 6. In the centre are the orbits of the two stars (red and green). Stable prograde orbits are possible in the outer (light gray) zone but not in the intermediate (medium gray) zone. Stable retrograde orbits however are possible in both zones.

Figure 6: The extended stable zone for retrograde orbits in the test system



Notes

In galactic dynamics, retrograde orbits (often referred to as 'counter-rotation') were once considered to be of mainly theoretical interest with no real examples, but are now known to occur for real, and in diverse forms, in numerous galaxies [8].

For an early study of retrograde orbits around a binary system see [9], and for some of the most recent and interesting work on retrograde circumbinary disks and accretion therein, see [10] and [11].

An online simulation of the test system is available [12].

Conclusion

Long-term numerical integration of a test system shows that the theoretical stable zone for circumbinary orbits is significantly larger for retrograde orbits than for prograde orbits. Retrograde circumbinary planetary orbits can be surprisingly close to the two stars, even in the generalised case where the stars have unequal mass and have eccentric stellar orbits.

Generalisation to other binary star systems

It is anticipated that for many other binary star systems, the theoretical circumbinary stable zone may similarly be significantly extended inwards, if retrograde circumbinary orbits are considered.
The increased size and inward extent of the circumbinary stable zone for retrograde orbits may in some systems result in overlap between it and the two circumstellar retrograde habitable zones.

References

[1] Backer, D.C.,
A pulsar timing tutorial and NRAO Green Bank observations of PSR 1257+12,
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[2] en.wikipedia.org/wiki/Circumbinary_planet

[3] Johannes Sahlmann, Amaury H. M. J. Triaud, David V. Martin,
Gaia's potential for the discovery of circumbinary planets
arxiv.org/abs/1410.4096

[4] This investigation uses as a test base the two stars of Nu Octantis, but does not include the proposed circumstellar massive planet.

[5] Matthew Holman, Paul Wiegert,
Long-Term Stability of Planets in Binary Systems,
arxiv.org/abs/astro-ph/9809315
See table 7 on page 11. For Nu Octantis, the value of μ (mass of secondary star divided by the the sum of the masses of primary and secondary stars) is about 0.263, and the stellar orbital eccentricity is about 0.236. Looking up those two values in the table, the approximate estimate of the critical semimajor axis for Nu Octantis is about 3.2 times the stellar orbital semimajor axis. The Nu Octantis binary has a semimajor axis of about 2.55 AU, therefore the approximate estimate of the critical semimajor axis for the test system examined here is about 8.1 AU.

[6] The initial parameters are listed in these units. The unit of mass is the mass of our Sun. The unit of distance is the average radius of our Earth's orbit (1 AU). The unit of velocity is the average orbital speed of our Earth (2π AU/ EY). The column labelled pX gives the X component of the initial position (the Y and Z components of all initial positions are zero). The column labelled vY gives the Y component of the initial velocity (the X and Z components of all initial velocities are zero).

[7] J. Eberle, M. Cuntz
On the reality of the suggested planet in the ν octantis system
iopscience.iop.org/2041-8205/721/2/L168
See Figure 2 therein, which illustrates the offset of a *circumstellar* orbital band within a binary.

[8] E.M.Corsini
Counter-Rotation in Disk Galaxies
arxiv.org/abs/1403.1263

[9] Victor Szebehely,
Theory of Orbits: The restricted problem of three bodies (1st edition, 1967),
See section 9.4.7: Synodic and sidereally retrograde periodic orbits around m1 and m2,

[10] C.J.Nixon,
Stable counteralignment of a circumbinary disc,
arxiv.org/abs/1204.4185

[11] C.J.Nixon, S.H.Lubow,
Resonances in retrograde circumbinary discs,
arxiv.org/abs/1501.07277

[12] Online HTML simulations of these orbits

Orbit example 1: www.orbsi.uk/space/simulator/simulator.htm?s=00061

Orbit example 2: www.orbsi.uk/space/simulator/simulator.htm?s=00060

Orbit example 3: www.orbsi.uk/space/simulator/simulator.htm?s=00059

Combined view: www.orbsi.uk/space/simulator/simulator.htm?s=00058