Co-precession

S.Edgeworth

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Introduction

This paper examines systems of coplanar nested highly-populated orbits in galaxies, and aims to explain how it is possible for every orbit within a set of nested orbits to have exactly the same precession rate.

Orbital streamlines

Throughout this paper, the orbital streamlines examined are of mode m=2. So the instantaneous shape of each streamline is an ellipse centred on the galactic centre. All the precession rate calculations are for m=2 streamlines.

Power-law fields

Simple axisymmetric power-law fields are examined, of the form $f \propto d^x$ (Equation 1)
which means that the force which attracts a unit mass towards the galactic centre varies in proportion to the distance from the galactic centre raised to some power **x** .

Definitions

The apsidal angle of an orbit is defined as the angle between two successive apocentres.
The radial period is defined as the time it takes for a star on the orbit to move from one apocentre to the next apocentre.
The axis ratio of an orbit is defined as the pericentre distance divided by the apocentre distance. (So an orbit with axis ratio = 1 is a circular orbit, and an orbit with axis ratio = 0 is a radial orbit).
A zero precession rate is defined as the precession rate of an orbit in the harmonic oscillator field, where the shape of the orbit, which is a centred ellipse, remains stationary in the inertial frame.
The precession rate of an orbit, defined as above, is given by $(\theta-\pi)/T_{rad}$
where θ is the apsidal angle, and T_{rad} is the radial period.
Prograde precession is in the same direction as the orbit.
Retrograde precession is in the opposite direction to the orbit.

Relationships

The theory is derived from these remarkably simple relationships:

For fields in between the keplerian and harmonic fields, i.e. fields with x in the range $(-2<x<1)$:
A. The apsidal angle is independent of orbit size.
B. The apsidal angle increases with increasing axis ratio
C. The apsidal angle is always greater than π, therefore precession is prograde.
D. For orbits of various sizes but all with the same axis ratio, the radial period increases with increasing orbit size.

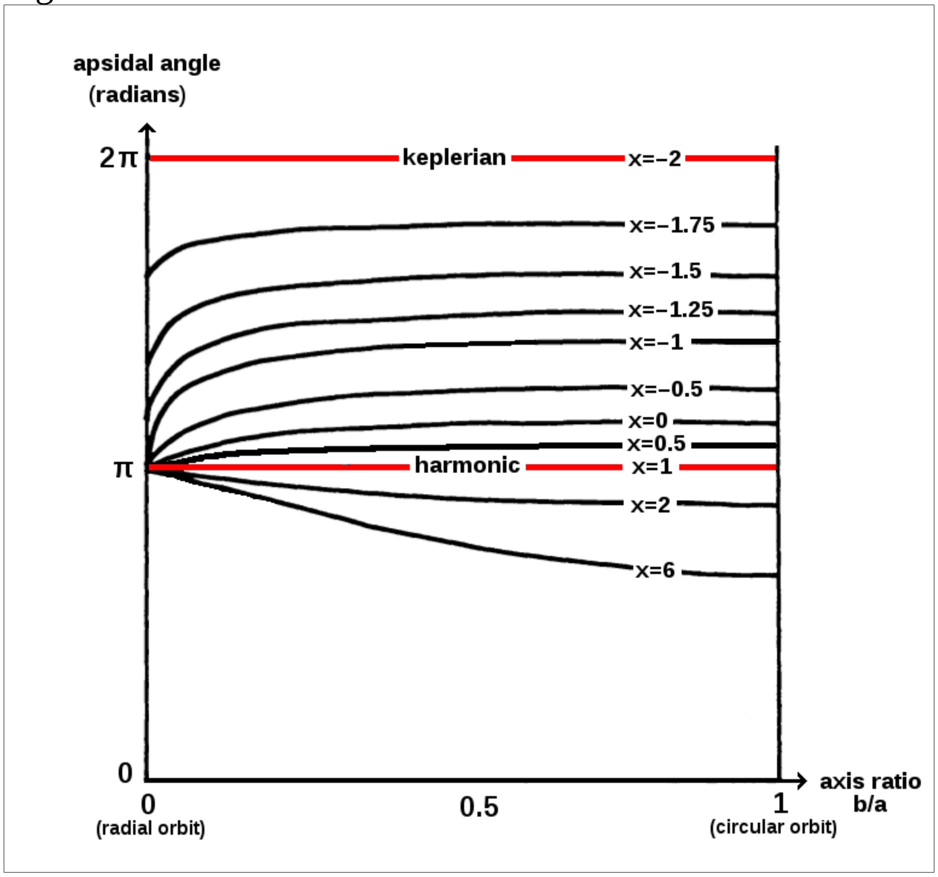
And for fields to the other side of the harmonic field, i.e. fields with x in the range $(1<x)$:
E. The apsidal angle is independent of orbit size.
F. The apsidal angle decreases with increasing axis ratio
G. The apsidal angle is always less than π, therefore precession is retrograde.
H. For orbits of various sizes but all with the same axis ratio, the radial period decreases with increasing orbit size.

Sources

Relationships D and H come from the equation $T_{rad} \propto d^{(2/(1-x))}$
(where T_{rad} is the radial period, d is the apocentre distance, and x is the exponent in equation 1), which describes how the radial period varies with orbit size when all orbits have the same axis ratio.

Relationships A B C E F and G are all based on Puel (1) and in particular on the remarkable shapes of the graph lines in the excellent diagram on page 6 of that work (1). Figure 1 below is directly based on that diagram. I have made various modifications, including relabelling the lines representing the various fields, as detailed in (2).

Figure 1:



This diagram is based on the diagram on page 6 of (1).

Definiton of type 0 fields

Type 0 fields are defined as fields which in equation 1 have x in the range (-2<x<1), i.e. fields intermediate between the keplerian field and the harmonic oscillator field (3).

Analysis of co-precessing orbits in type 0 fields

In a type 0 field, two nested orbits are created, one slightly larger than the other, but both having the same axis ratio, in the range (0<<axisratio<<1).
From rule A, both orbits have the same apsidal angle.
From rule C, both orbits precess in the prograde direction.
From rule D, the larger orbit has the longer radial period.
Therefore the the larger orbit will precess in the prograde direction less rapidly than the smaller orbit.
To achieve co-precession, it is necessary to make the the apsidal angle of the larger orbit greater than the apsidal angle of the smaller orbit.
From rule B, the only way to increase the apsidal angle is to increase the axis ratio (to make the orbit more circular). Therefore if the the orbits are to co-precess, the larger orbit must have a greater axis ratio (must be more circular) than the smaller orbit.
So to achieve co-precession of a set of many nested orbits, the orbits must have axis ratios which increase (the orbits must become more circular) with increasing orbit size.

Definition of type 1 fields

Type 1 fields are defined as: fields which in equation 1 have x in the range (1<x), i.e. fields on the other side of the harmonic oscillator field.

Analysis of co-precessing orbits in type 1 fields

In a type 1 field, two nested orbits are created, one slightly larger than the other, but both having the same axis ratio, in the range (0<<axisratio<<1).
From rule E, both orbits precess in the retrograde direction.
From rule G, both orbits have the same apsidal angle.
From rule H, the larger orbit has the shorter radial period.
Therefore the the larger orbit will precess in the retrograde direction more rapidly than the smaller orbit.
To achieve co-precession, it is necessary to make the the apsidal angle of the larger orbit greater (closer to π) than the apsidal angle of the smaller orbit.
From rule F, the only way to increase the apsidal angle is to decrease the axis ratio (to make the orbit less circular). Therefore if the the orbits are to co-precess, the larger orbit must have a smaller axis ratio (must be less circular) than the smaller orbit.
So to achieve co-precession of a set of many nested orbits, the orbits must have axis ratios which decrease (the orbits must become less circular) with increasing orbit size..

Computer analysis

The characteristic outline shapes , and the pitch angles and azimuthal extents of spiral arms, are from the orbit diagrams in (4) and (5) where complete sets of co-precessing orbits are obtained by computer. (The analysis of the trailing sense of spiral arms will be detailed in a future paper).

Conclusion

In various simple axisymmetric power-law gravity fields, it is possible to create a theoretical galactic component consisting of many nested and highly-populated orbits, with the important property that every orbit of the component co-precesses. Two separate ranges of fields produce two distinctive types of component, differing greatly in their typical characteristics, as follows:

Definition of type 0 component:
Generated by some type 0 fields, with x in the range ($-2<x<1$).
Axis ratio of the orbits increases (orbits become more circular) with increasing orbit size.
All orbits of the component have the same precession rate.
Precession (pattern speed) is prograde relative to orbital direction.
Orbital period increases with increasing orbit size.
Spiral arms, if present, typically have small pitch angle and large azimuthal extent.
Spiral arms, if present, are always trailing relative to the orbital motions.
Outline shape is typically fairly circular.

Definition of type 1 component:
Generated by type 1 fields, with x in the range ($1<x$).
Axis ratio decreases (orbits become more elliptical) with increasing orbit size.
All orbits of the component have the same precession rate.
Precession (pattern speed) is retrograde relative to orbital direction.
Orbital period decreases with increasing orbit size.
Spiral arms, if present, typically have large pitch angle and small azimuthal extent.
Spiral arms, if present, are always trailing relative to the orbital motions.
Outline shape is typically fairly elliptical.

In following papers these simple theoretical results will be applied to make specific predictions about the kinematics and structure of several real galaxies.

References

(1) Puel, F.,
The rotation number of bounded orbits in a central field
1983CeMec..29..255P

(2) The vertical axis, labelled in the original diagram as rotation number, is relabelled here as apsidal angle (rotation number is simply apsidal angle divided by 2π). The lines representing the various fields, originally labelled by the equations for the potentials, are relabelled here by the exponents of the equations for the gradients of the potentials. The lines for several fields have been removed, and the line for the field x=0.5 has been added.

(3) Blitzer, L.
Hyper-Elliptic Orbit
1988CeMec..42..215B

(4) Equalisation of precession rates of galactic orbits (Part 1)
www.orbsi.uk/space/research/se/pdf/equalisation-precession-galactic-orbits-1.pdf

(5) Equalisation of precession rates of galactic orbits (Part 2)
www.orbsi.uk/space/research/se/pdf/equalisation-precession-galactic-orbits-2.pdf