Galactic bars in power-law fields

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The precession of galactic orbits was investigated using C++ software which integrates galactic orbits in various power-law fields, and which calculates the precession rate of each orbit.

A power-law field may be written as $\mathbf{f} \propto \mathbf{d}^{\mathbf{x}}$ (equation 1)
which means that the force **f**, which attracts a unit mass towards the centre of attraction, varies in proportion to the distance **d** from the centre raised to some power **x** . The power-law fields examined here are assumed to be axisymmetric. Fields are described here by the value of **x.** So for example where a field is described below as (x=−2), the equation for the field may be obtained from equation 1 by simply by replacing x with the value −2.

The aim is to find the range of power-law fields in which elongated galactic bars may be constructed. An elongated bar is defined here as a galactic bar in which the orbit streamlines have ellipticities which increase with increasing streamline size. The bar is assumed to consist of nested highly-populated coplanar m=2 streamlines. The strict requirements are applied that the streamlines must be non-intersecting and must all co-precess.

Initially, power law fields intermediate between the keplerian field and the harmonic oscillator field [1] were investigated. This is the range of fields which would conventionally be expected in a galaxy. It was found that there is a sub-range of these fields, in which it is possible to construct sets of nested m=2 orbit streamlines which exactly co-precess. For example, this was successfully done for the (x=0.5) field, and also for the (x=0.25) field. Those sets of nested co-precessing orbit streamlines have ellipticities which decrease with increasing streamline size, and resemble galactic disks with grand design spiral arms of small pitch angle. But they do not resemble the elongated bars we are seeking to generate here.

It was found to be impossible to construct an elongated galactic bar, in any power-law field intermediate between the keplerian field (x=−2) and the harmonic oscillator field (x=1). This is because, for all fields in the range (−2<x<1), co-precession of a set of nested m=2 streamlines requires that the streamlines have ellipticities which decrease with increasing streamline size.

Therefore the search was extended to fields beyond those conventionally expected in a galaxy. Power-law fields in the range (1<x) were investigated. In other words, power-law fields situated to the far side of the harmonic oscillator field. It was found that for a sub-range of these fields, it was also possible to construct nested sets of m=2 streamlines which co-precess. Significantly, in this range of fields, co-precession requires that the streamlines have ellipticities which increase with increasing streamline size. For example, this was successfully done for the (x=2) field [2]. The resulting set of co-precessing nested m=2 streamlines closely resembles an elongated galactic bar.

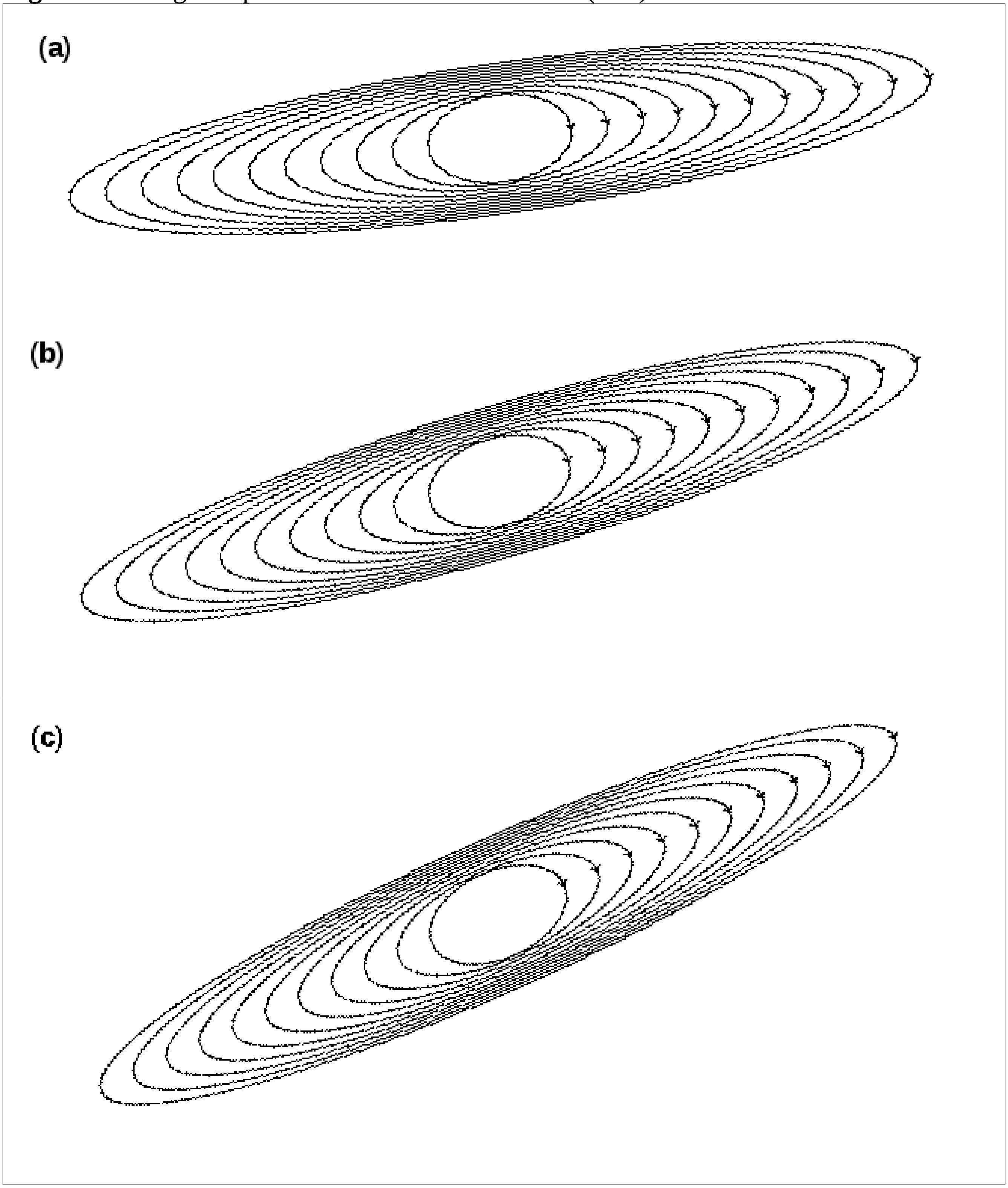
A galactic bar in the (x=2) field is illustrated in figure 1. It should be noted that the ellipticity gradient is not an optional feature, it is essential because it is what makes the streamlines all precess at the same rate. Without this precisely-tuned ellipticity profile, the streamlines would all precess at different rates, become intersecting, and the bar structure would be quickly destroyed.

Now some remarkable properties of elongated bars in power-law fields may easily be deduced:
They require that the field is in the range (1<x).
Orbital period decreases with increasing streamline size.
Velocities measured on the minor axis increase more than linearly with distance from the galactic centre.
Pattern rotation is retrograde relative to the direction of the orbits.

The pattern rotation of an elongated bar in the (x=2) field is illustrated in figure 1. The arrows indicate the directions of the orbits. Each of the three pictures shows an instantaneous view of the streamlines, in the inertial frame. Picture (a) shows the bar at time t=0, and pictures (b) and (c) show the bar at successively later times. The pattern rotation is in the opposite direction to the orbital motions.

The axis ratios of the streamlines are listed in [2], and range from 0.6 for the inner streamline to 0.172 for the outer streamline. Each of these axis ratios was carefully calculated iteratively using C++ software so that the streamlines co-precess.

Figure 1: retrograde pattern rotation of a bar in the (x=2) field



A java applet simulation is available [3] which illustrates in moving graphics the bar described above, which may be viewed in the inertial frame, or in a frame co-rotating with the pattern rotation of the bar.

The starting parameters for orbit integration software are provided in figure 2. The initial position (x, y) and velocity (vx, vy) are listed for one particle of each of eleven streamlines. The system of units used is chosen so that a circular orbit of radius r=1 has orbital velocity $v_{circ}=1$, angular velocity $\omega_{circ}=1$, and orbital period $T_{circ}=2\pi$.

Figure 2: Starting parameters for bar orbits in the (x=2) field

x	**y**	**vx**	**vy**
0.2	0	0	0.0485
0.3	0	0	0.0595
0.4	0	0	0.0727
0.5	0	0	0.08696
0.6	0	0	0.1008
0.7	0	0	0.1157
0.8	0	0	0.1298
0.9	0	0	0.144
1.0	0	0	0.1581
1.1	0	0	0.173
1.2	0	0	0.1868

In another investigation, as yet unpublished, an elongated galactic bar with uniform precession was likewise constructed in the (x=1.5) field.

The range of power-law fields, in which an elongated bar with uniform precession may be constructed, extends continuously through the range (1.5<x<2) and extends to some extent beyond that range in both directions.

Also available is an HTML5 simulation [4] which illustrates that similar bar orbits are obtained in the (x=1.5) field. Inertial view or co-rotating view may be selected.

Conclusions

An elongated galactic bar was defined as a bar in which the streamlines have ellipticities increasing with streamline size. Systems of co-precessing nested m=2 orbital streamlines were constructed in various axisymmetric power-law fields. For fields intermediate between the keplerian field and the harmonic-oscillator field, the resulting sets of nested streamlines do not resemble elongated galactic bars. However, for fields beyond the harmonic-oscillator field, the results do resemble elongated galactic bars. An example of a set of co-precessing bar streamlines was provided for the (x=2) field, and the orbital starting parameters listed. Elongated bars constructed in axisymmetric power-law fields have the following remarkable properties:

(A) The orbital periods of the streamlines of the elongated bar decrease with increasing streamline size.
(B) The pattern rotation of the elongated bar is retrograde relative to its orbits.

A seperate paper will extend this investigation to multiple bars (bars nested within bars) and their relative speeds of pattern rotation. A further paper will examine the dynamical coupling which may occur between an elongated bar (with the remarkable properties described above) and a galactic disk.

References

[1] Blitzer, L.
Hyper-Elliptic Orbits
1988CeMec..42..215B

[2] Edgeworth, S.
Equalisation of precession rates of galactic orbits (Part 2)
www.academia.edu/3508228

[3] Java applet integration of bar orbits in the (x=2) field
www.orbsi.uk/space/g/gal-exp-two.htm
(Requires a browser which can display java applets).

[4] HTML5 integration of bar orbits in the (x=1.5) field
www.orbsi.uk/space/simulator/simulator.php?s=00032

Version history
v.1: 29 Sep 2013
v.2: 07 Jan 2015 updated link to java simulation
v.3: 02 Dec 2017 added HTML5 simulation