The Inner Lindblad Resonance in Galactic Dynamics

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This analysis starts with the reasonable requirement that if a galactic component (for example a bar, or a disc) has a uniform pattern speed, and consists of a set of coplanar nested orbits, then all of those orbits must have the same precession rate.

In (1) and (2) it was shown, in each of two example galactic fields, that by applying a precise ellipticity profile it is possible to construct a system of nested coplanar orbits in which every orbit shares an equal precession rate. Both papers include links to Java applets which illustrate the orbits in motion.

A galactic component was constructed in the **$f \propto \sqrt{d}$** field, in which every orbit has the same precession rate (1).

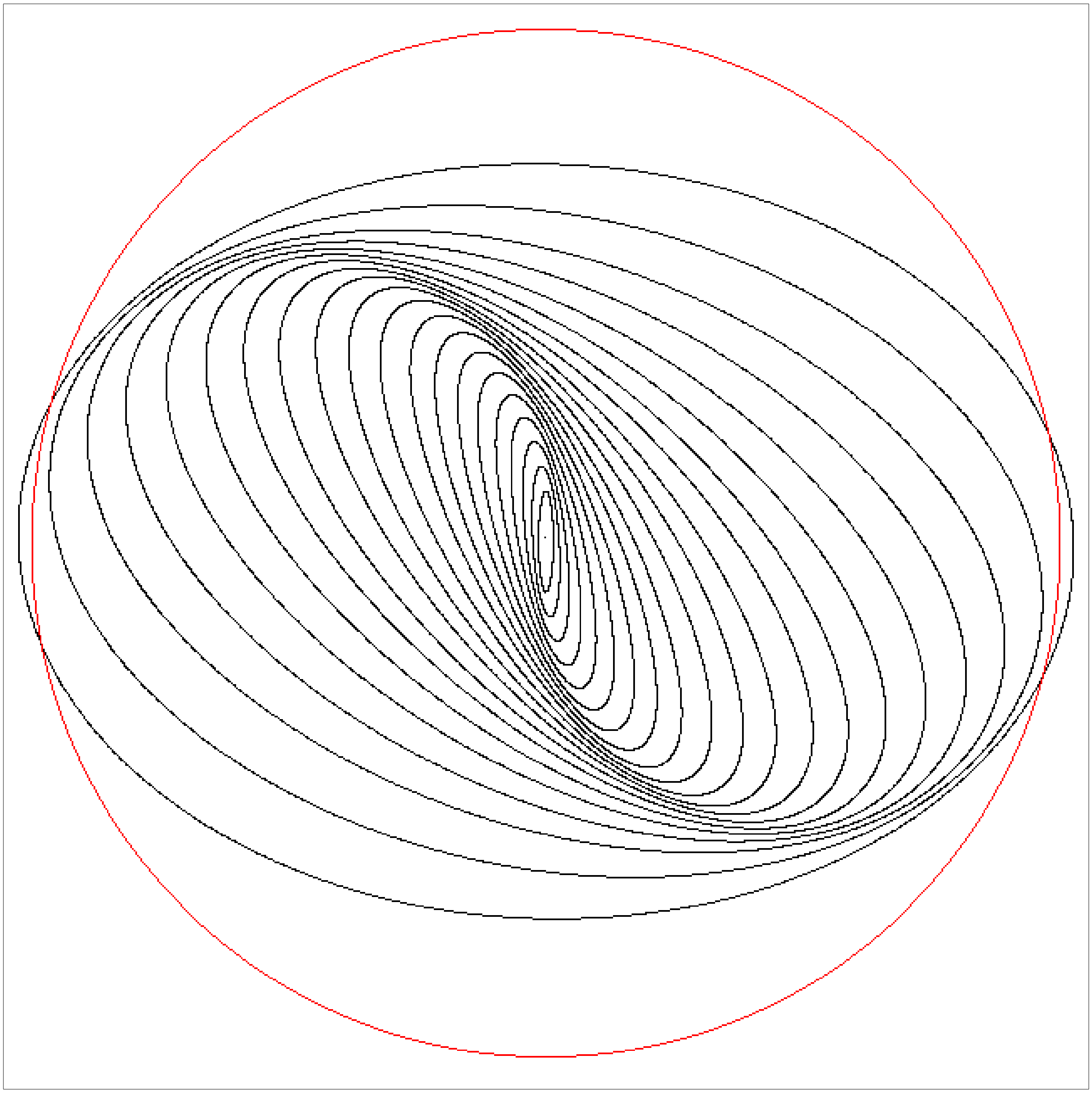
The aim in the current paper is to answer the question:
"Which of these orbits is at the component's inner Lindblad resonance?" (3).

Figure 1 shows the calculated orbits, in a frame which co-rotates at the component's pattern speed, and in an alignment which produces kinematic density waves (spiral arms). The orbits are in the clockwise direction. The uniform precession (pattern speed) is in the clockwise direction. The spiral arms are trailing.

Textbooks on galactic dynamics commonly treat the inner Lindblad resonance as corresponding to a circular or nearly-circular orbit.

The hypothetical circular or nearly-circular inner lindblad resonance was calculated, and is shown as a red circle in figure 1. It is immediately clear that it does not match any of the orbits in this component. The red orbit intersects and collides with some of the existing orbits. So applying the textbook conception of the inner-lindblad resonance, which treats it as a nearly-circular orbit, would indicate that none of the orbits in this component are at the component's inner Lindblad resonance.

Figure 1: Orbits with equal precession rates



However, we now drop the restriction which treats the inner Lindblad resonance as a circular or nearly-circular orbit. Instead we allow for the possibility that an orbit with considerable ellipticity may be in inner Lindblad resonance.

When we examine the orbit of any star in any of these orbits in figure 1, it is clear that the frequency at which the star encounters a density wave is exactly equal to the star's radial frequency. This is one form of the definition of the inner Lindblad resonance (4). Therefore every orbit in this component is in inner Lindblad resonance with the component.

Equally, a careful consideration of the very different galactic component presented in (2), will show that all of those orbits are in inner Lindblad resonance.

This deduction, about the inner Lindblad resonance in co-precessing systems, may be generalised to any galactic system of co-precessing nested co-planar orbits, in any field, by noting that the terms "co-precessing at the component pattern speed" and "in inner Lindblad resonance" are equivalent in meaning. An orbit which precesses at the component precession rate will have a radial frequency equal to the frequency at which it encounters a density wave, and therefore will be in inner Lindblad resonance.

Conclusion

It is proposed that in a galactic component (a bar or a disc) consisting of co-precessing nested coplanar orbits, every orbit is in inner Lindblad resonance with the component.

References

(1) Equalisation of precession rates of galactic orbits (Part 1),
http://www.orbsi.uk/space/research/se/pdf/equalisation-precession-galactic-orbits-1.pdf

(2) Equalisation of precession rates of galactic orbits (Part 2),
www.orbsi.uk/space/research/se/pdf/equalisation-precession-galactic-orbits-2.pdf

(3) Throughout the current paper, the term "inner Lindblad resonance" refers to the **first** inner Lindblad resonance.

(4) Sellwood, J. A., Dynamics of Disks and Warps, (Section 2.4),
ned.ipac.caltech.edu/level5/March11/Sellwood/Sellwood2.html