Proposed solution of the winding problem in galactic dynamics

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Binney and Tremaine [1)] examine the winding problem and conclude that "density waves can resist the winding process better than material arms" and quantify this by estimating that "the wave pattern winds up more slowly than the material arm by a factor of about five". They then present the interesting challenge "if only we could find a way to adjust the slow drift rates of all the orbits to a common standard". It is that challenge which directly inspired this work, and which this paper aims to answer.

This paper embraces the density wave theory of spiral arms. However it seems clear that dividing the winding problem by five is insufficient. The aim here is to reduce the winding problem theoretically to zero, by requiring that all of the orbits (which contribute to the spiral density waves) must share one common apsidal precession rate.

The co-precession theory, from which the proposed solution of the winding problem is derived, is described theoretically in [4], and quantitively by computer calculations in [5]. In all potentials, other than the single special case of the harmonic oscillator potential [2], nested coplanar non-intersecting orbit streamlines of different sizes will have different radial periods (the time interval between successive apocentres). So it follows that, if the orbit streamlines all have the same apsidal angle (the angle between successive apocentres), they will inevitably all have different precession rates.

However, a remarkable characteristic of various potentials (excluding the two special cases of the harmonic oscillator potential and the keplerian potential), is that adjusting the ellipticity of a given orbit will alter its apsidal angle, as demonstrated in the remarkable paper [3], and will therefore directly alter its precession rate
And so a set of nested elliptic orbit streamlines may in some fields achieve co-precession by means of a precisely tuned ellipticity profile. The other requirement is that the resulting orbits with their various ellipticities must remain non-intersecting. The stable configuration is achieved when the differences in the radial periods of the orbits are exactly compensated for by the differences in the ellipticities and hence the apsidal angles of the orbits, so that all the orbits co-precess.

Figure 2 in [3] clearly shows that, in potentials intermediate between the keplerian potential and the harmonic oscillator potential, the precession rate of an orbit may be increased, by reducing the ellipticity of the orbit and thereby increasing the apsidal angle.

So a set of nested orbit streamlines may, in some fields in this range, be made to co-precess by giving the orbit streamlines successively smaller ellipticities with increasing orbit size, providing that the adjusted orbit streamlines remain non-intersecting.

Figure 3 in [5] lists the parameters of 25 orbit streamlines calculated numerically by computer, in a power-law field intermediate between the keplerian field and the harmonic-oscillator field. Of those orbit streamlines, 23 are co-precessing. Their semi-major axes range from 0.001 to 1.05, and their axis ratios (minor axis divided by major axis) range from 0.032 to 0.715, yet all these orbits have the same precession rate. Figures 4 and 5 in [5] illustrate 20 of those co-precessing orbit streamlines.

The effect of the decreasing ellipticity on precession rate exactly balances the effect of the increasing orbit size on precession rate, producing identical precession rates for the nested orbit streamlines and eliminating the winding problem.

It is interesting that in solar system dynamics, a broadly equivalent solution, namely a monotonic ellipticity gradient, has been proposed to explain the co-precession of the orbit streamlines of eccentric planetary rings [6].

Conclusion

It is proposed that the winding problem (in a galactic disk with grand spiral density waves of small pitch angle and large azimuthal extent) may be largely eliminated by a precisely generated ellipticity profile which produces co-precession all the orbit streamlines of the disk. It is predicted that in this type of galactic disk, the orbit streamlines will have ellipticities decreasing with increasing orbit size.
A galactic disk calculated by software was studied, in which the nested orbit streamlines have a huge range of sizes, with semi-major axes ranging from 0.001 to 1.05, and it was shown that a precisely tuned ellipticity profile, with the axis ratios in that example ranging from 0.032 for the innermost orbit to 0.715 for the outermost orbit, produces co-precession of all the orbit streamlines, effectively solving the winding problem.

References

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