Pulsating retrograde invariant loops around eccentric binaries

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Part 1

If a binary star system has circular stellar orbits, it is relatively easy to deduce the orbits of circumbinary planets or disk particles. However, if the stellar orbits are elliptic, then the circumbinary orbits are considerably more complicated [4] .

The test system consists of two star of equal masses, but with a large stellar orbital eccentricity = 0.333. This introduces a time-varying component in the combined gravitational field of the two stars.

A test planet (or disk particle) is placed at initial distance from the barycentre = 1.25 times the stellar semimajor axis, in conjunction, at stellar apoapsis. The planet is given a retrograde circumbinary orbit, with an initial velocity iteratively calculated to produce an invariant loop, by a technique similar to that used for prograde orbits in [5] and [6].

Specially adapted numerical integration software calculates the size and shape of the invariant loop at stellar phases from 0.0 to 0.9 in increments of 0.1.

The first video [1] loops through those nine phases, showing how the size and shape of the loop pulsates with a period equal to the stellar orbital period. The orbit of the planet (or alternatively the set of orbits of the disk streamline) is retrograde, anticlockwise along the pulsating loop.

This is just one example of a retrograde pulsating invariant loop, and it is anticipated that the exact characteristics of the pulsation will be different in other examples.

Part 2 (18 April 2015)

The ripples and almost-straight sections at some phases in the first video [1] are real, they are not software artifacts. However, a more careful optimisation of the loop, not shown here, significantly reduces the occurence of these features.

It has been known for a considerable time that orbits in the elliptic restricted three body problem may be successfully treated as motions along pulsating invariant loops, see for example [4]. The aim of the current paper is simply to facilitate visualisation, by calculating some loops and presenting them as videos.

The second video [2] illustrates a further four pulsating invariant loops around the same binary star system (m1=m2, e=0.333). As before, the circumbinary orbits are retrograde, anticlockwise along the loops.

The inner pair of loops, although they have different modes in their pulsations, are sufficiently synchronised to prevent them intersecting.

The average orbital periods of the four illustrated loops are very approximately 1.23 and 1.4 and 2.3 and 2.6 times the stellar orbital period. Somewhere between the two illustrated pairs of loops, there are indications that there may be a gap, where invariant loops cannot be constructed. A preliminary and unproven guess is that the gap may be centred on the -2:1 orbital period resonance. There appears to be a similar gap somewhere outside the outer pair of illustrated loops, where a preliminary and unproven guess is that it may be centred on the -3:1 resonance. It seems reasonable to presume that there will be further gaps.

Numerical integration for 74000 stellar orbital periods indicates that all 4 illustrated loops are stable. The method of calculating the exact size and shape of an invariant loop, at various phases of its pulsation cycle, from a single optimised orbit, will be described in a later addendum.

Part 3 (20 April 2015)

The outer group of loops in the second video is next expanded to make a group of 5 loops which is illustrated in the third video [3]. This is around the same binary star system (m1=m2, e=0.333). Again the circumbinary orbital directions around each loop are anticlockwise, the opposite direction to the orbits of the two stars. (Please note that the apparent motion of the displayed dots on some loops is a irrelevant strobe effect of the video generation method and should be ignored).

The five loops have mean orbital periods equal to very approximatly 2.18 and 2.30 and 2.44 and 2.60 and 2.80 times the stellar orbital period. It seems likely that it will be possible to add further loops internally within this group, with orbital periods intermediate between those values.

It was found to be not possible to continuously extend this particular group of loops much further inwards or outwards. A preliminary guess is that this particular group of loops may be bounded at its inner and outer limits, by orbital gaps centred on the -2:1 and -3:1 resonances.

In the strong pulsation of the rotating gravitational field, the 5 loops pulsate in synchronisation. If each of the loops in this group (the 5 illustrated loops plus further intermediate loops added in between them) is imagined to be populated by a stream of many small particles distributed all around its perimeter, then the streams are estimated (although not proven) to be mutually non-intersecting, and the difference between the velocity of one stream and its neighbour streams is small. Therefore any random collisions occuring between particles are likely to be at small relative velocities, which, if other conditions are also right, may favour accretion into larger particles.

The inner two loops (purple and light blue) in the second video [2] are closer to the stars than the -2:1 orbital resonance, with orbital periods approximately equal to 1.23 and 1.4 times the stellar orbital period. These two loops differ from each other in their modes of pulsation and therefore belong to two different groups. This investigation has not yet determined whether or not either of these groups can be expanded into a set of nested mutually non-intersecting loops

The way in which a group of retrograde circumbinary loops may pulsate in synchronisation, so that even in the pulsating field, they do not intersect each other, is similar to the situation for prograde circumbinary orbits which has been described in [7].

In the prograde case, there is a big gap between the two stars and the inner boundary of the circumbinary stable zone. In the retrograde case however, the circumbinary stable zone extends much closer to the two stars, and the gap is smaller. This means that retrograde circumbinary loops can be stable in a region where the magnitude of the pulsation of the stellar gravitational field is huge, and the resulting pulsations of the retrograde circumbinary loops are easily presented visually in the videos here.

Provisional conclusions so far

If a binary star system with e=0.333 and m1=m2 has a hypothetical retrograde circumbinary disk, then the inner part of the disk is likely to be banded, and one particular band was identified above, comprising a set of orbital loops intermediate between the -2:1 and -3:1 resonances (and possibly bounded by orbital gaps at those resonances). It is provisionally estimated (but not proven) that this group of loops is mutually non-intersecting, in which case the relative velocity of random particle collisions would be low. If so, then this may facilitate over a long period of time the accretion of the numerous small disk particles of this particular group of loops, resulting in the in-situ formation of one or more retrograde circumbinary planets, with orbital period greater than 2 but less than 3 times the stellar orbital period, and with semimajor axis greater than 1.72 but but less than 2.02 times the stellar semimajor axis.

Closer to the two stars, in the even more ambitious region inside the -2:1 resonance, many stable planetary orbits are possible, and three examples have been illustrated, see video[1], and the two inner loops in video [2]. This investigation intends later to determine whether or not a group of loops exists, inside the -2:1 resonance, with the crucial property of mutual non-intersection, and this will determine whether or not the in-situ accretion of planets within this ambitious inner band is possible.

The region further from the two stars, outside the -3:1 resonance, has in this paper not yet been investigated, however it seems likely that there will be further groups of mutually non-intersecting loops, where in-situ accretion will be possible, perhaps seperated by further gaps at resonances for example the -4:1 resonance.

It was demonstrated in [7] that (excluding an unstable region relatively close to the two stars) the accretion of planets may occur in a prograde circumbinary disk, just as easily as in a circumstellar disk around a single star. It is proposed here that accretion of planets may likewise occur just as easily in a hypothetical retrograde circumbinary disk, and moreover that the zone where accretion may occur extends much closer to the two stars in a retrograde circumbinary disk than in a prograde circumbinary disk. Around the hypothetical binary system examined here (m1=m2, e=0.333), it seems likely that in-situ accretion may be theoretically possible even in the range of semimajor radii between about 1.7 to 2.0 times the stellar semimajor axis, corresponding to orbital periods in the range intermediate between 2 and 3 times the stellar orbital period, as well as at greater distances.

Part 4 (28 April 2015)

An online simulation which shows the five optimised orbits, from which video 3 was generated by a further software layer, is available at [8].

In the simulation, the traced paths of the five orbits give the incorrect impression that the orbits collide with each other. In fact these planets never collide. The five planets travel along five orbital loops which pulsate in unison with each other and with the stellar orbits, so that they never intersect, as shown in video 3. Careful observation of the simulation demonstrates that the pink planet always passes the green planet on the inside, and the green planet always passes the white planet on the inside, and so on. This is the remarkable way in which an eccentric binary star system, even though it has a strongly pulsating gravitational field, is theoretically able to support stable retrograde circumbinary planets (or bands of disk particles), even at this proximity to the two stars.

References

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