A theoretical example of locked resonance of the orbit of an S-type planet in 3:1 ratio to the orbit of its host binary star system

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The possibility that a planet in an S-type orbit in a binary star system might have an orbital period resonant with a simple fraction of the orbital period of the host binary star system was suggested by Ramm et al (1). They found radial velocity residuals for the nu octantis system which suggested, amongst the possible explanations, the possible presence of an S-type planet completing exactly 5 planetary orbits per 2 stellar orbits. That work was extended by Eberle and Cuntz (2) and by other researchers (3) (4) (5) (6).

This study describes a theoretical planetary orbit which is resonantly locked so that it completes exactly 3 planetary orbits per 1 orbit of the binary star system. The system described here has stellar mass ratio and stellar orbital eccentricity approximating those of the nu octantis system (1). It should be emphasised however that the orbit described here is theoretical, and of different orbit ratio to the possibly real planet described in the above references.

The orbits were integrated using a second-order symplectic n-body integration method. As a safety check, the orbits were also seperately integrated with completely different software using an explicit symmetric multistep method. The results of the two methods are in full accord.

First the starting parameters for the three bodies are listed so that orbit may be replicated in any n-body integration software. (The orbits are coplanar therefore all Z values are zero).

[Primary star]
Mass = 1.4 SM
PosX = - 0.829286842105 AU
PosY = 0
VelX = 0
VelY = (0.178629233587 * 2 * pi) AU/EY

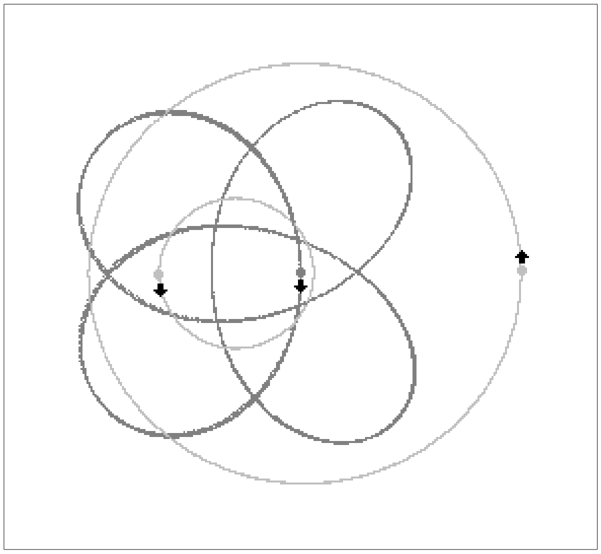
[Secondary star]
Mass = 0.5 SM
PosX = 2.32200315789 AU
PosY = 0
VelX = 0
VelY = (- 0.500161854036 * 2 * pi) AU/EY

[Planet]
Mass = 0.0038184 SM
PosX = 0.4 AU
PosY = 0
VelX = 0
VelY = (1.2 * 2 * pi) AU/EY

Abbreviations: SM = the mass of our sun, AU = astronomical unit, EY = earth year, pi = 3.14159...

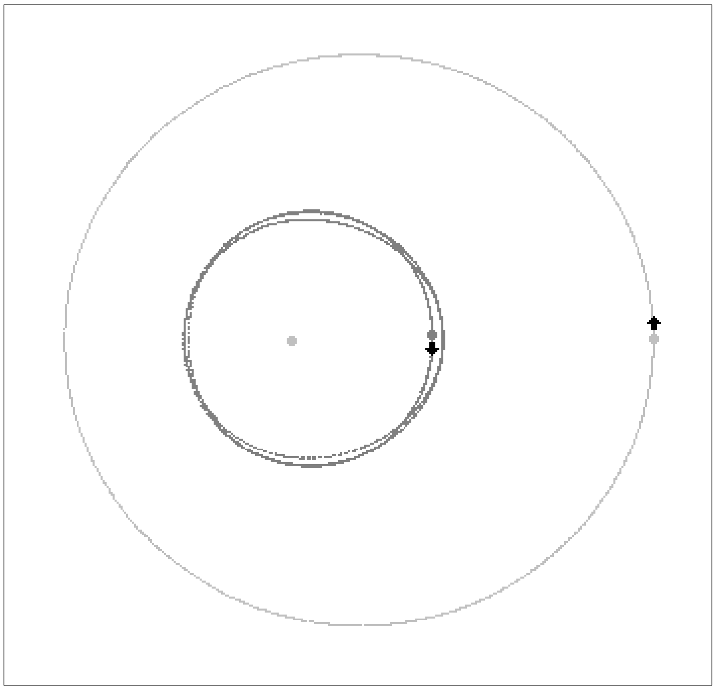
The planetary orbit, when viewed in the inertial frame, repeats the same four-lobed path once per 3 planetary orbital periods, as illustrated in diagram 1.

Diagram 1:



The planetary orbit, when viewed in a frame which is fixed on the primary star, occupies an approximately circular band, as illustrated in diagram 2. This band is not centred on the primary star, but is offset. This offset of S-type orbits was previously described by (2).

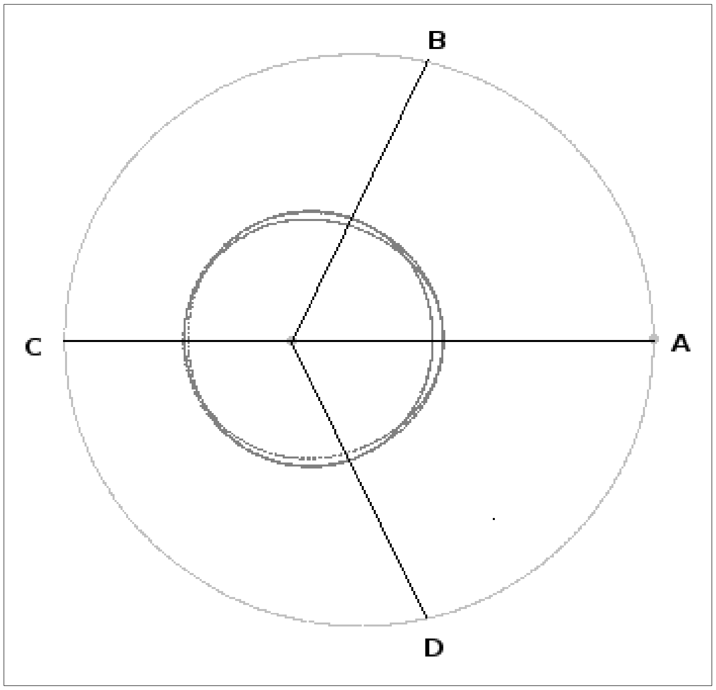
Diagram 2:



The eccentricity of a perturbed orbit may be treated as comprising two parts, the *forced* eccentricity due to perturbation, and the *free* eccentricity (7). The orbit described here has been optimised, so that it has almost zero *free* eccentricity. S-type planetary orbits which are optimised to have almost zero *free* eccentricity, and which are in simple orbit ratio to the host star system, do not fill their orbital band in the short-term, but simply repeat a defined path within the band. The planetary orbit described here follows a path which it repeats once per three planetary orbits, leaving the remainder of the orbital band, in the short-term, unfilled, as illustrated in diagrams 2 to 7.

The planet has 4 conjunctions (when it passes directly in between the two stars) per 3 planetary orbital periods. The 4 conjunctions are labelled A B C and D in diagram 3, viewed in a frame which is fixed on the primary star.

Diagram 3:



When the planetary orbit is viewed in a frame which co-rotates with the binary star system, the path which the planet follows repeatedly once per three planetary orbital periods can be most clearly seen.

The position of the planet, at the each of the four conjunctions A, B, C, and D, is illustrated in diagrams 4, 5, 6 and 7 respectively. These four diagrams are all viewed in a frame which is fixed on the primary star and which co-rotates which the star system. Note that in this viewing frame, the primary star is stationary, and the secondary star moves back and forth, between apoapsis and periapsis, along a straight line.

Diagram 4:

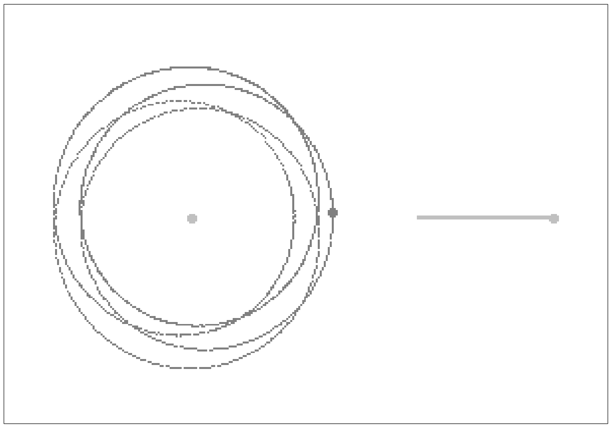


Diagram 5:

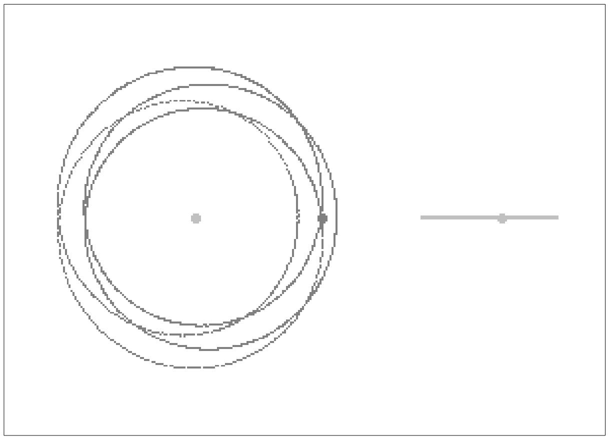


Diagram 6:

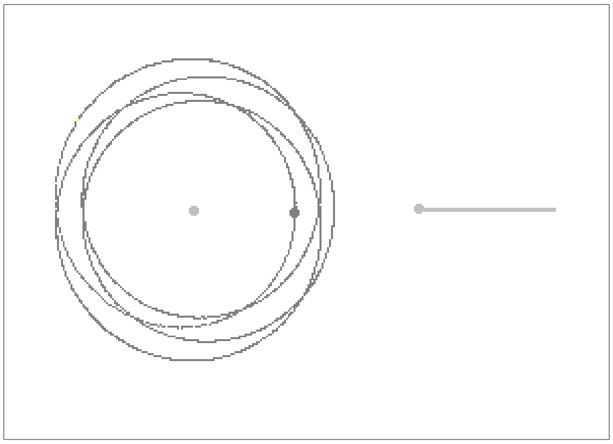
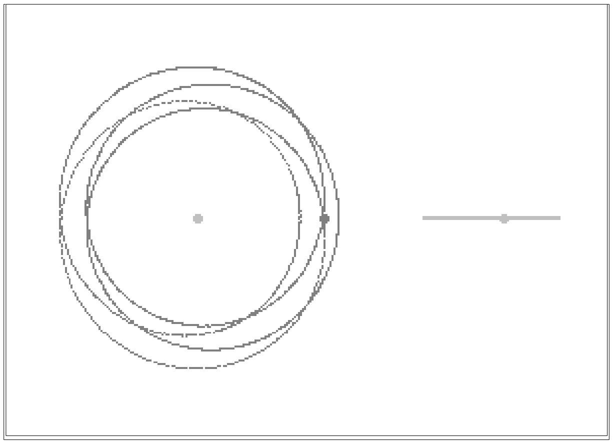


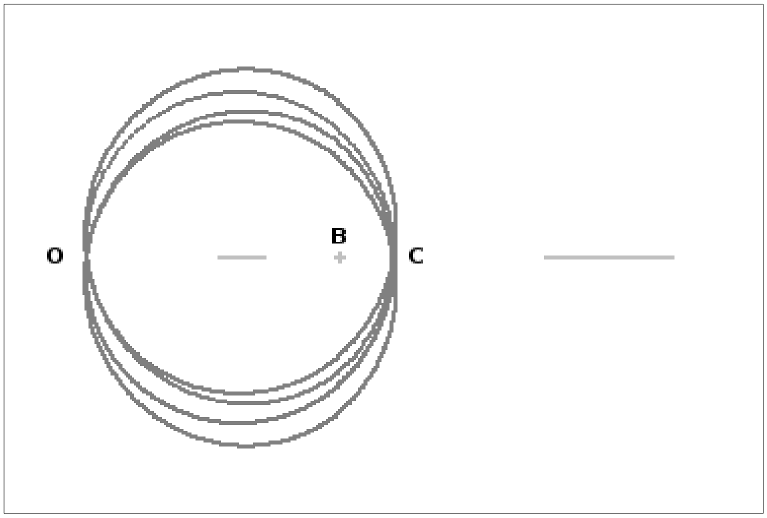
Diagram 7:



Next the orbit is illustrated in a frame of reference which is co-rotating with the star system, and which is centred on the system barycentre, see diagram 8.

Viewed in this frame of reference, the motion of the parent star is back and forth along a straight line (the shorter of the two straight lines in diagram 8), with the two ends of the line being reached at stellar apoapsis and periapsis respectively. Similarly the motion of the other star is back and forth along a straight line (the longer of the two straight lines in diagram 8). The curved line is the orbit of the planet. The system barycentre is marked with a small cross and labelled B.

Diagram 8:



The planet experiences, during each stellar orbital period, four conjunctions (when the bodies are exactly aligned in the order star–planet-star) and four oppositions (when the bodies are exactly aligned in the order planet-star-star).

Now two interesting observations may be made:
(a) The distance of the planet from the system barycentre at each of the four conjunctions is approximately the same (distance CB in diagram 8).
(b) The distance of the planet from the system barycentre at each of the four oppositions is approximately the same (distance OB in diagram 8).

Having described the general characteristics of this theoretical planetary orbit, next the behaviour of the planetary and stellar orbits when integrated over a longer time is summarised as follows.
These three features all precess together at the same rate and in the same direction:
(a) The line of apses of the stellar orbit.
(b) The set of the four lines of conjunction.
(c) The shape of the planetary orbit.
The precession is in the same direction of the motion of the two stars, and at a rate which is numerically estimated to be very approximately 0.1 degrees per stellar orbital period.

An HTML5 simulation of the orbit is available (8), which enables this theoretical orbit to be watched as a live integration, and is viewable in a choice of four frames of reference.

It is concluded that the orbit of the planet described here (which is theoretical not real) is in 3:1 resonance with the orbit of the star system. And as the theoretical star system modelled here has stellar mass ratio and stellar orbital eccentricity approximating those of the nu octantis system, it follows that a stable resonant 3:1 circumstellar orbit would theoretically be possible in a hypothetical binary star system similar to Nu Octantis.

References:

(1) Spectroscopic orbits for K giants β Reticuli and ν Octantis: what is causing a low-amplitude radial velocity resonant perturbation in ν Oct?
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(2) On the reality of the suggested planet in the ν octantis system
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(3) The Stability of the Suggested Planet in the nu Octantis System: A Numerical and Statistical Study
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(5) Testing a hypothesis of the ν Octantis planetary system
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(6) Dynamical analysis of the ν Octantis planetary system
Goździewski K., Słonina M., Rozenkiewicz A., Migaszewski C.
Proceedings of the workshop "Orbital Couples: Pas de Deux in the Solar System and the Milky Way". Held at the Observatoire de Paris, 10-12 October 2011. Editors: F. Arenou, D. Hestroffer. ISBN 2-910015-64-5, p. 97-102

(7) Solar System Dynamics
Murray, Dermott
The concepts of forced and free eccentricities are described on several pages, for example p.309.

(8) HTML5 orbit simulation: www.orbsi.uk/space/simulator/simulator.htm?s=00043

Version notes:
Version 5: 07 Jan 2015 (updated the link to the java animation at new web address)
Version 6: 03 May 2015 (added link to HTML5 simulation)
Version 7: 29 Nov 2017 (removed link to Java simulation)