Retrograde circumbinary orbits around simple binary star systems

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27 April 2015

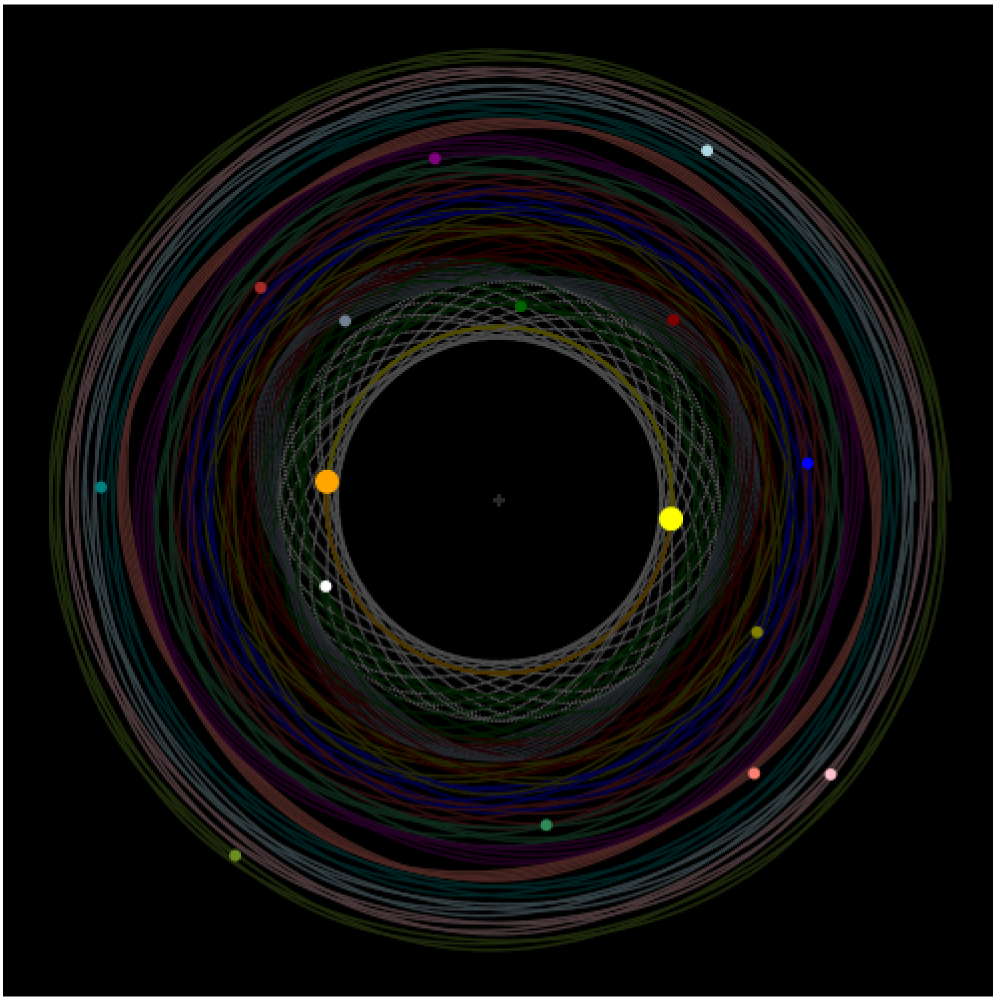
Sets of retrograde circumbinary orbits, like those illustrated here, were already calculated by Möller in 1924 [1] , and have been further examined by various other authors such as Henon [2] and Szebehely [3].

This paper improves on the description and illustration of nested circumbinary orbits which was already provided in [4], by showing a larger quantity of orbits, with more even orbital spacing. The results are presented as an online simulation [5]. The illustrations here are screenshots of the moving graphics of the online simulation.

Software is used to numerically integrate a test system comprising two stars and 14 circumbinary planets. The two stars have equal masses and circular stellar orbits, and the planets have negligible mass. The planets are started at various distances from the binary stars, and are given retrograde circumbinary orbits with almost zero free eccentricity.

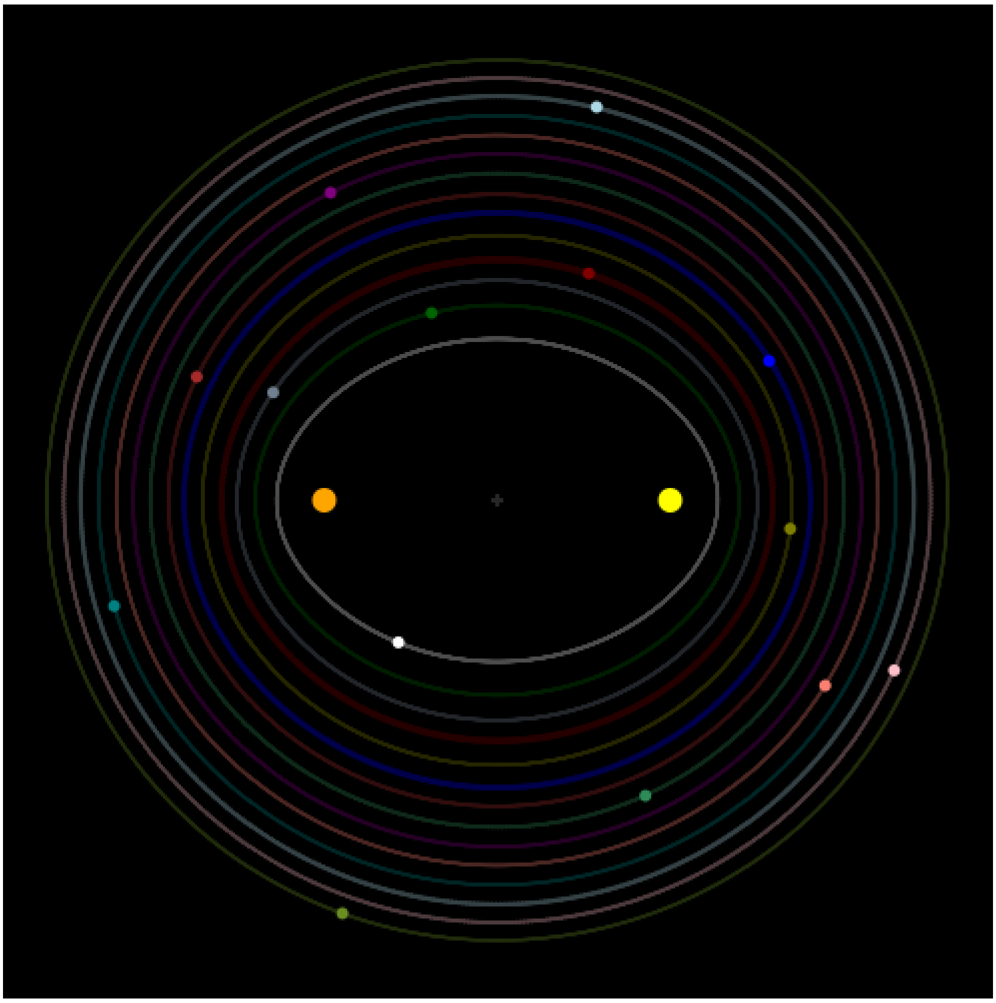
The resulting orbits are illustrated in the inertial frame in Figure 1, traced for a duration of about 5 stellar orbits It can be seen that the optimised planetary orbits all have significant deviations from circular paths, the deviations being greatest for the innermost planet, and least for the outermost planet.

Figure 1: Orbits viewed in the inertial frame



Initially it may be tempting to assume from figure 1 that the orbits intersect. However these orbits are in fact non-intersecting. In figure 2 the orbits are shown in a viewing frame which co-rotates with the orbits of the two stars, so that the two stars appear stationary, and it can be seen that the orbits are neatly nested ovals which never intersect.

Figure 2: Orbits viewed in the corotating frame



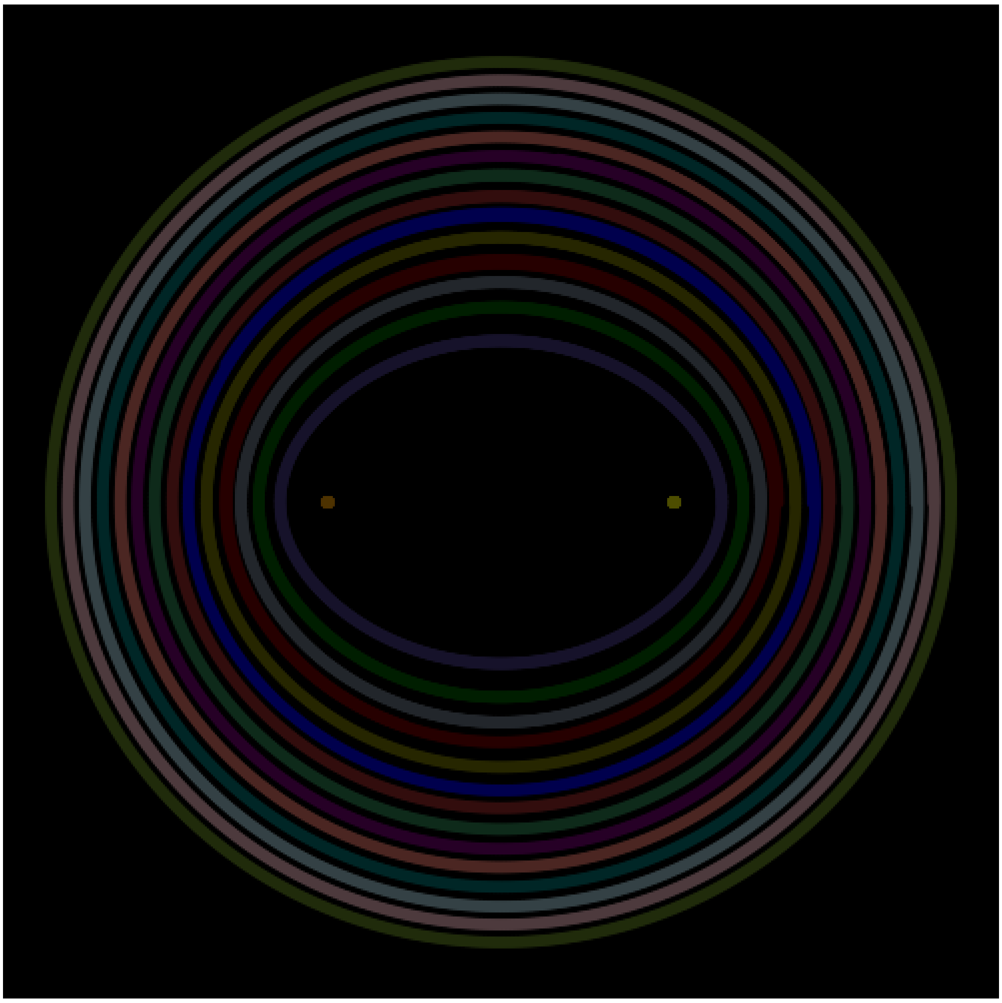
Details of the orbits are listed in table 1. Some of the innermost orbits have orbital periods even shorter than the orbital period of the two stars. The innermost orbit passes closer to the barycentre than do the two stars.

Table 1: Details of the resulting orbits

Orbit	Ratio of semimajor axis to stellar semimajor axis	Orbital eccentricity	Ratio of orbital period to stellar orbital period
Stellar	1	0	1
Planet 1	0.55	0.156	0.32
Planet 2	0.63	0.108	0.41
Planet 3	0.69	0.084	0.48
Planet 4	0.75	0.067	0.55
Planet 5	0.81	0.053	0.63
Planet 6	0.86	0.043	0.71
Planet 7	0.92	0.036	0.79
Planet 8	0.97	0.030	0.86
Planet 9	1.03	0.025	0.95
Planet 10	1.08	0.021	1.03
Planet 11	1.13	0.018	1.12
Planet 12	1.18	0.015	1.20
Planet 13	1.23	0.013	1.29
Planet 14	1.29	0.011	1.38
(many further orbits may be added)			

Next, each planet is hypothetically removed, and replaced by a large quantity of small particles of negligible mass, distributed around the orbit as viewed in the corotating frame. This converts each planetary orbit into a streamline of many small particles. Actually each small particle in a given streamline has its own unique orbit. However the many orbits of a given streamline all have exactly the same orbital period, and are coordinated so that in the corotating frame they always follow the oval path of the streamline. Representative streamlines of the resulting hypothetical circumbinary disk are illustrated in figure 3.

Figure 3: Circumbinary disk streamlines viewed in the corotating frame



In the illustrations and online simulation, only the innermost orbits have been shown. The set of orbits may be extended considerably outwards by adding further larger orbits. The starting parameters for all the orbits examined here are available on request.

The results illustrated here are scalable to any simple binary star system in which the two stars have circular orbits and equal masses. The slightly more complicated circumbinary orbits around binary stars with unequal stellar masses, and with eccentric stellar orbits, will be examined in seperate papers.

References

[1] Möller JP
Die einfach periodischen, retrograden Bahnen um die beiden endlichen Massen im problème restreint, mit retrograder absoluter Bewegung.
Astronomische Nachrichten, 221, 81-90, (1924)
adsabs.harvard.edu/full/1924AN....221...81M
For example "Bahn 18" is similar to orbit 9 in the current paper, and "Bahn 19" is slightly larger than orbit 1 in the current paper, also he calculates some even smaller orbits , and two orbits larger than those illustrated here. Möller numerically integrated many of these orbits using a triple-core analog processor [6].

[2] Henon M
Exploration numérique du problème restreint. II. Masses égales, stabilité des orbites périodiques (1965)
adsabs.harvard.edu/abs/1965AnAp...28..992H
See "classe m orbits"on page 13, and figure 16.

[3] Szebehely V
Theory of orbits: the restricted problem of three bodies (1967)
See "class m orbits" in section 9.4.7 and section 9.10, and figure 9.48

[4] Edgeworth S
Theoretical Orbits of Planets in Binary Star Systems (2001)
www.orbsi.uk/space/research/se/pdf/theoretical-orbits-planets-binary-star-system.pdf
See illustration of nested retrograde circumbinary orbits on page 12

[5] Online simulation of 14 retrograde circumbinary orbits
www.orbsi.uk/space/simulator/simulator.php?s=00055
Requires a modern browser which fully supports the most recent HTML standards.

[6] The triple core analog processor consisted of Möller, Vinter Hanson, and Løkkegaard, using slide-rules.