The stable zone for P-type planetary orbits in a binary star system with equal masses and zero orbital eccentricity

S Edgeworth, 2012

The stability zone for P-type planetary orbits has previously been investigated, for a range of stellar mass ratios and stellar orbital eccentricities, in various studies including those listed in references (1) (2) (3) (4) (5) (6) (7) (8) (9). This study investigates the stability zone in the special case in which the two stars have equal mass and zero orbital eccentricity.

The system is assumed to have the following properties:
The two stars have equal masses.
The stellar orbital eccentricity is zero.
The planet has negligible mass.
The planetary orbital inclination is 180 degrees (i.e. the orbit is coplanar and retrograde).
The eccentricity of the planetary orbit is minimised.
There are no tidal effects and no other complicating factors.

N-body gravitational integration software was used to test a large number of orbits. Each test started with the planet exactly at conjunction with the stars, and with the motion of the planet exactly perpendicular to the line of conjunction.

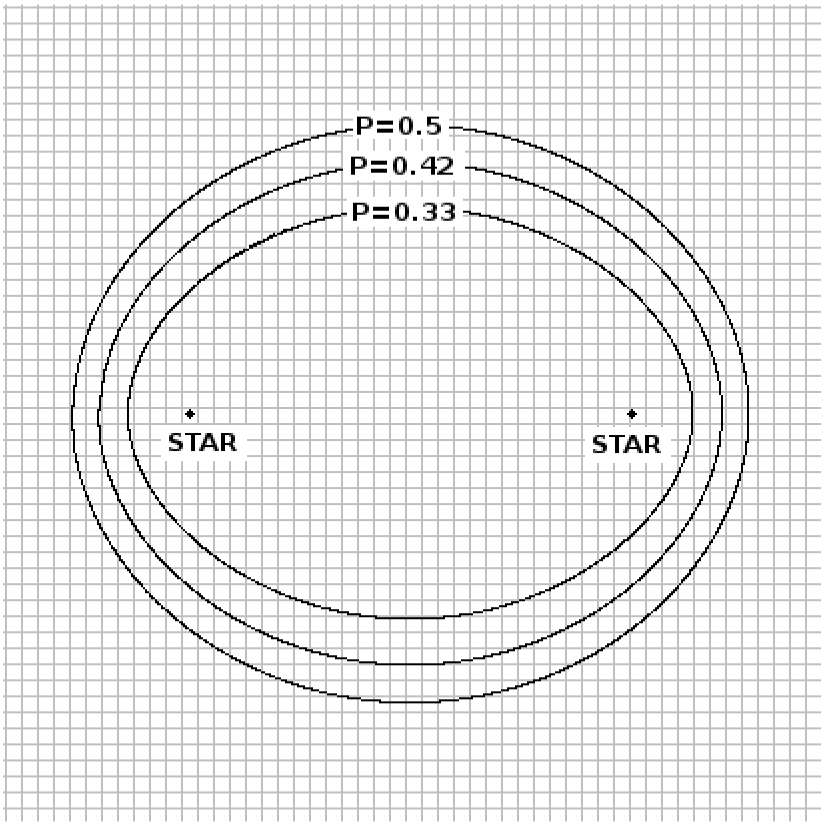
A range of initial positions of the planet was tested, at various distances of the planet from the system barycentre (at conjunction). The unit of distance used here is defined as: 1 unit = the distance seperating the two stars.

For each initial position of the planet, a range of initial velocities of the planet was tested, each resulting in a different orbit. The initial velocity was then fine-tuned until an orbit was obtained which has almost no excess eccentricity beyond that caused by perturbation. (10).

In practice this is implemented in software by adjusting the initial velocity until the orbit forms a simple closed path when viewed in a frame which co-rotates with the star system. So for each initial position, from the range of orbits obtained, one optimised orbit can be found, which forms a simple closed path when viewed in the co-rotating frame.

The next diagram illustrates three examples of these optimised P-type orbits, with period ≈ 0.5, period ≈ 0.42, and period ≈ 0.33 of the stellar orbital period, viewed in a frame which co-rotates with the star system.

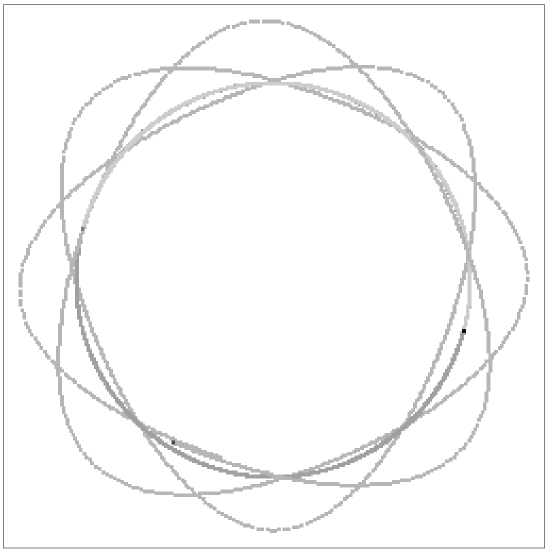
Diagram 1: Three examples of optimised P-type orbits, viewed in a frame which co-rotates with the star system



The orbit with the shortest period (and therefore of smallest spatial dimensions) found in this study, has orbital period ≈ 0.33 of the stellar orbital period. When viewed in the co-rotating frame the orbit is approximately oval in shape. At conjunctions the planet is at distance ≈ 0.637 from the barycentre, and midway between conjunctions the planet is at distance ≈ 0.46 from the barycentre. (The unit of distance is defined as 1 unit = the stellar seperation).

This orbit was tested for a duration of 10000 planetary orbits. Diagram 2 shows the orbit of the planet (and the circular orbit of the stars), viewed in the inertial frame and traced for 3 planetary orbits, after the completion of 10000 planetary orbits. The survival of, and the preservation of the form and dimensions of the planetary orbit over the duration of 10000 planetary orbits, are indications that the orbit is stable.

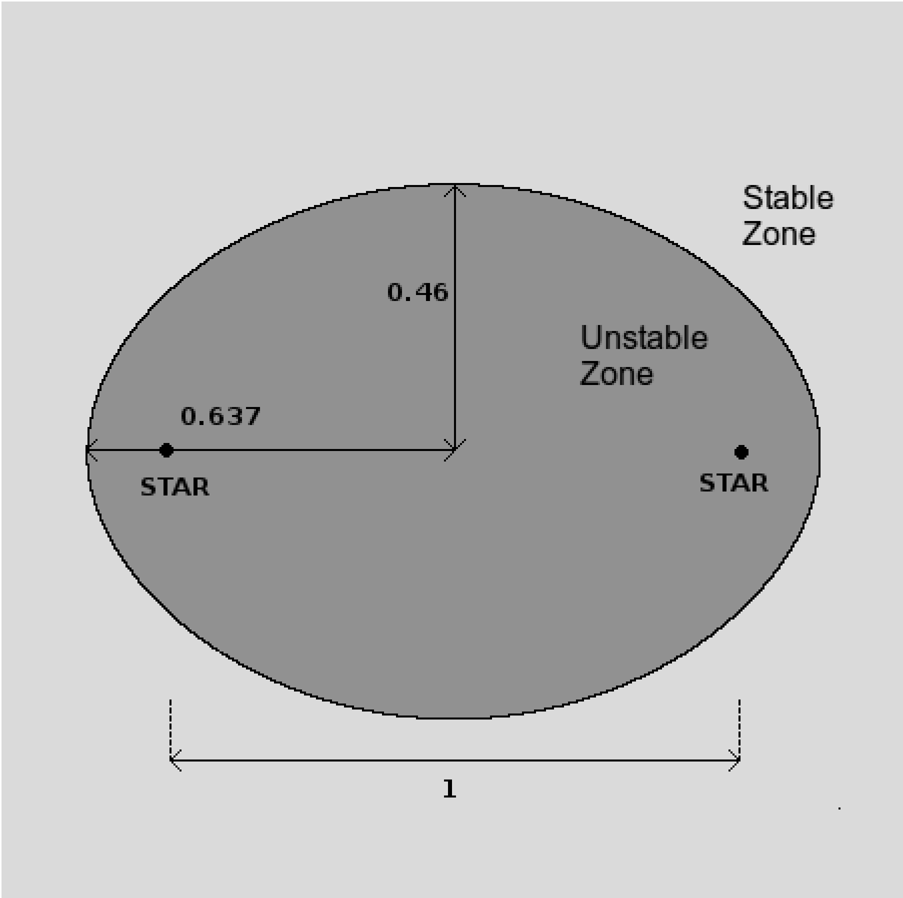
Diagram 2: P-type orbit with period ≈ 0.33, viewed in the inertial frame



The inner boundary of the P-type stability zone may be spatially defined, in the co-rotating frame, by the dimensions of the smallest stable orbit. So from the dimensions of the orbit described above, the following estimate of the stability zone boundary is obtained.

The next diagram illustrates the estimated inner boundary of the stable zone for P-type orbits, in a system where the two stars have equal mass and zero orbital eccentricity, and with the other restrictions listed near the beginning of this study. The view is in the co-rotating frame, and the boundary of the stable zone is approximately an oval with semi-major axis ≈ 0.637 times the stellar seperation, and semi-minor axis ≈ 0.46 times the stellar seperation. The unstable zone is shaded dark gray, and the stable zone is shaded light gray.

Diagram 3: An estimate of the P-type stable zone (in a system where the two stars have equal mass and zero orbital eccentricity) viewed in a frame which co-rotates with the star system



This estimate of the stability zone boundary is approximate. A precise definition of the boundary of the stability zone, in binary systems as defined above, will depend on the exact definition of stability chosen, and on the extent to which the strict assumptions (such as the absence of tidal effects) are relaxed.

The set of initial parameters, sufficient to replicate in n-body software the smallest orbit described here, is listed on the next page.

An HTML5 simulation of a hypothetical binary star system with 14 planets in P-type orbits close to the inner boundary of the stability zone, is available at the web address listed below (11). The simulation can be viewed in the inertial frame, and in the co-rotating frame. The smallest orbit in the simulation is the orbit described above, with orbital period equal to a third of the stellar orbital period. The largest orbit in the simulation has an orbital period equal to approx 1.38 times the stellar orbital period. All of these orbits would be instantly unstable if they prograde. These orbits are stable because they are retrograde.

Initial parameters: P-type orbit with period ≈ 0.33 of the stellar period

```
[Body1]
Name= StarA
Mass= 0.5 SM
PosX= -0.5 AU
PosY= 0
PosZ= 0
VelX= 0
VelY= (0.5 * 2 * pi) AU/EY
VelZ= 0

[Body2]
Name= StarB
Mass= 0.5 SM
PosX= 0.5 AU
PosY= 0
PosZ= 0
VelX= 0
VelY= (-0.5 * 2 * pi) AU/EY
VelZ= 0

[Body3]
Name= Planet
Mass= 0
PosX= -0.6370341 AU
PosY= 0
PosZ= 0
VelX= 0
VelY= (-2.398413511 * 2 * pi) AU/EY
VelZ= 0
```

Abbreviations: SM = the mass of our sun. AU = astronomical unit. EY = earth year. pi = 3.14159...

References:

(1) Eberle, Cuntz, Musielac (2008)
Orbital Stability of Planets in Binary Systems
(Exoplanets: Detection, Formation and Dynamics, Proceedings of the IAU, IAU Symposium, Vol.249, p.507-510)

(2) Funk (2002)
Stability of P-type Orbits in Binaries
(Hvar Observatory Bulletin, vol.26, no.1, p.57-61)

(3) Holman, Wiegert (1999)
Long-Term Stability of Planets in Binary Systems
(Astronomical Journal, Vol.117, Issue 1, p.621-628)

(4) Musielac, Cuntz, Marshall, Stuit (2005)
Stability of Planetary Orbits in Binary Systems
(Astronomy and Astrophysics, Vol.434, Issue 1, Apr IV 2005, p.355-364)

(5) Musielac, Cuntz, Marshall, Stuit (2008)
Stability of Planetary Orbits in Binary Systems (erratum)
(Astronomy and Astrophysics, Vol.480, Number 2, March III 2008, p.573)

(6) Pilat-Lohinger, Funk, Freistetter, Dvorak (2002)
Stability of P-type Orbits in Double Stars
(Proceedings of the First European Workshop on Exo-Astrobiology, 16-19 Sept 2002, Graz, p.547-548)

(7) Pilat-Lohinger, Dvorak, Funk, Bois, Freistetter (2003)
On the dynamical stability of planets in double stars
(Proceedings of the Conference on Towards Other Earths, 22-25 Apr 2003, Heidelberg, p.543-548)

(8) Pilat-Lohinger, Funk, Dvorak (2003)
Stability limits in double stars
(Astronomy and Astrophysics, Vol.400, Number 3, Mar IV 2003)

(9) Pilat-Lohinger, Dvorak (2007)
On the Stability of Extra-Solar Planetary Systems
(Topics in Gravitational Dynamics: Lecture Notes in Physics Vol.729, 2007, p.209-232)

(10) Airy (1838) treated the eccentricity of a perturbed orbit as comprising two parts, in ("Gravitation" in Penny Cyclopaedia of the Society for the Diffusion of Useful Knowledge, Vol.XI, p.386, paragraph 134)

(11) HTML5 simulation of 14 P-type planetary orbits around a binary star system.
www.orbsi.uk/space/simulator/simulator.htm?s=00055

Version notes
v.4 07 Jan 2015 Updated link to java animation at new web address.
v.5 29 Nov 2017 Removed java applet, and added HTML5 simulation.