Theoretical Dynamics of Nested Galactic Bars

S. Edgeworth
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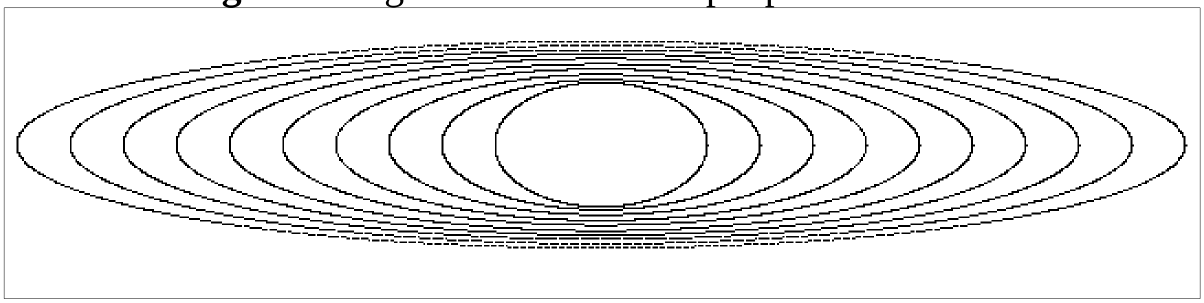
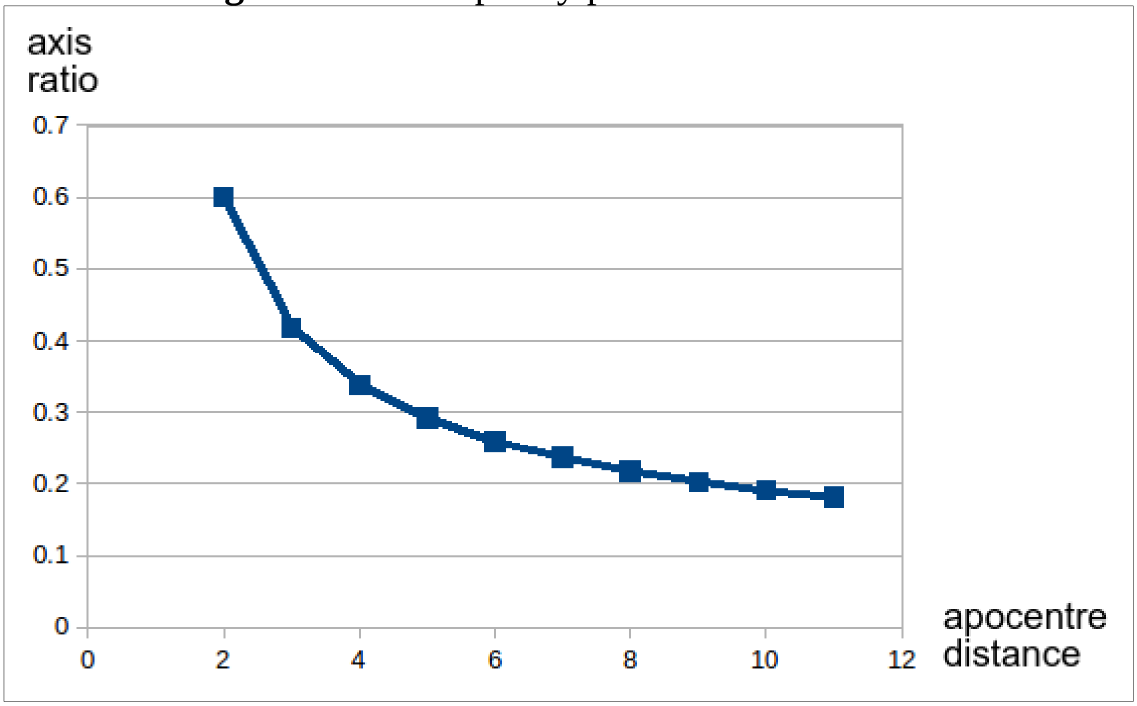
The dynamics of an elongated bar in a simple power-law galactic field were investigated in [1]. It was shown that the bar has uniform pattern rotation in the opposite sense to the direction of its streamline orbits.

Figure 1: A galactic bar in a simple power-law field



The uniform pattern rotation is achieved because the streamlines of the bar have an ellipticity profile which precisely equalises the precession rates of all the streamlines. Each streamline is slightly less circular than the slightly smaller streamline immediately inside it.

Figure 2: The ellipticity profile of the outer bar



This paper extends that investigation to find out what dynamical system may be formed by the particles which exist in the region between the inner limit of the bar, and the galactic centre.

The system of units used is chosen so that a circular orbit of radius r=1 has orbital velocity v_{circ}=1, angular velocity ω_{circ}=1, and orbital period T_{circ}=2π. The axis ratio of a streamline is defined as its pericentre distance divided by its apocentre distance. The total acceleration acting on each particle is assumed to be towards the galactic centre, and proportional to its distance from the galactic centre raised to some power x, and this paper specifically examines the field in which x=2.

It is not possible to extend the outer bar further inwards by adding further, smaller streamlines. This is because there are no further (smaller) streamlines which satisfy both of the two strict conditions: that they must have the same pattern rotation speed as the outer bar, and that they must be non-intersecting with the existing streamlines of the outer bar.

Therefore the particles, which exist in the region between the galactic centre and the inner boundary of the outer bar, may form a separate system of streamlines, which has a different pattern rotation rate (precession rate) than the outer bar.

The simple power law gravity field which we are examining here is scale-free. This means that the inner system of streamlines (the inner bar) will be a scaled-down self-similar copy of the outer bar.

The innermost streamline of the outer bar has apocentre distance = 2, and axis ratio = 0.6, therefore it has pericentre distance = 1.2.

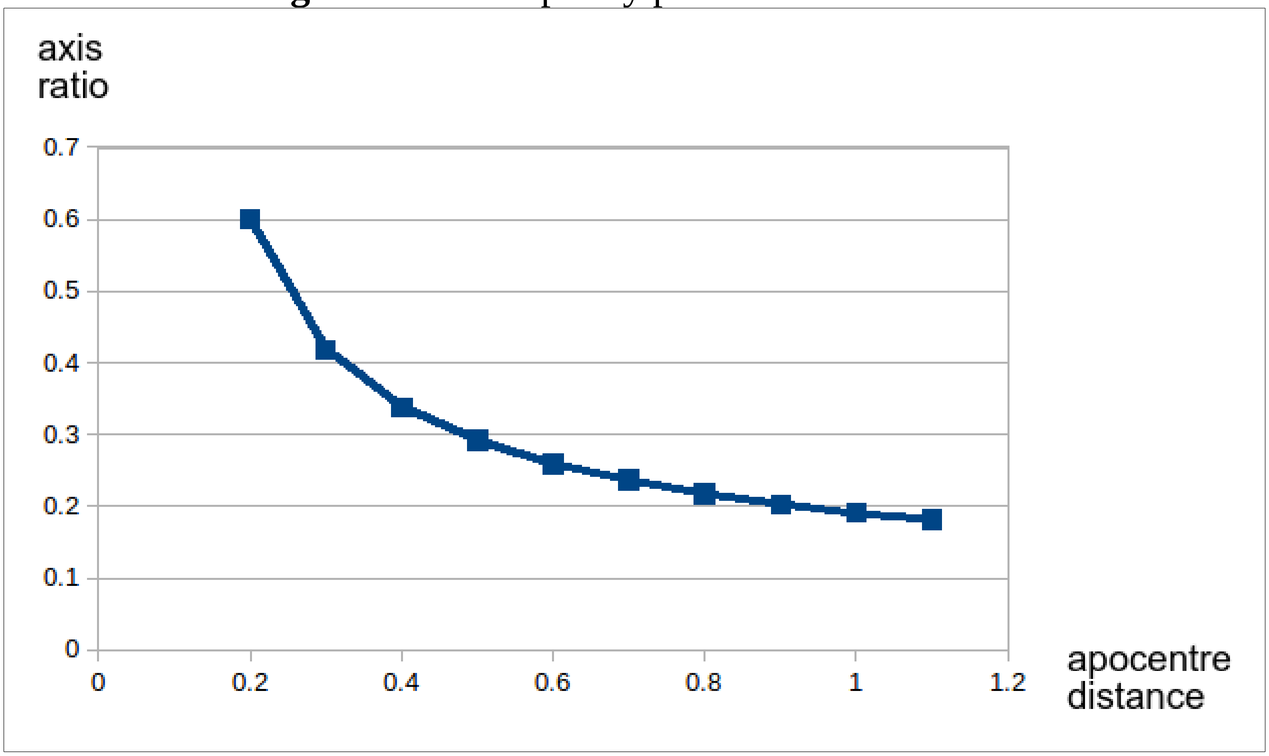
The inner bar will necessarily have a pattern rotation rate (precession) different to that of the outer bar. It is essential that the outermost streamline of the inner bar must never intersect the innermost streamline of the outer bar, and this must be satisfied for all orientations of the inner bar relative to the outer. bar. Therefore the apocentre distance of the outermost streamline of the inner bar must be less than 1.2.

Therefore the apocentre distance of the outermost orbit of the inner bar is here set = 1.1, which means that the inner bar is a self-similar copy of the outer bar with the scale ratio k = 10.

The scale ratio is defined as: the apocentre distance of a streamline in the outer bar, divided by the apocentre distance of the equivalent streamline (the streamline with the the same axis ratio) in the inner bar.

The inner bar has the same ellipticity profile as the outer bar, except that all distances are scaled down by the scale ratio k=10, as shown in figure 3.

Figure 3: The ellipticity profile of the inner bar



Ratio of absolute pattern rotation rates of outer and inner bars

Using:

$d_{outer} =$ the apocentre distance of a selected streamline of the outer bar

$d_{inner} =$ the apocentre distance of the equivalent (same axis ratio) streamline of the inner bar

$k =$ the scale ratio $= \frac{d_{outer}}{d_{inner}}$

$x =$ the exponent in the simple power-law function for acceleration toward the centre

$\Omega_{outerbar} =$ the absolute angular rate of pattern rotation of the outer bar

$\Omega_{innerbar} =$ the absolute angular rate of pattern rotation of the inner bar

The ratio of the absolute pattern rotation rates of the two bars is given by:

$$\frac{\Omega_{outerbar}}{\Omega_{innerbar}} = k^{\frac{x-1}{2}}$$

The values of x and k are then inserted:

$$x = 2$$

$$\frac{\Omega_{outerbar}}{\Omega_{innerbar}} = k^{\frac{1}{2}}$$

$$k = 10$$

$$\frac{\Omega_{outerbar}}{\Omega_{innerbar}} = \sqrt{10}$$

Which, for the example system examined here, gives the result:

$$\Omega_{innerbar} = 0.316 * \Omega_{outerbar}$$

Dynamics with orbits of inner bar in same sense as orbits of outer bar

There are two possibilities for the dynamics.

In the first possibility, the orbits of the inner bar are in the **same** sense as the orbits of the outer bar.

The resulting pattern rotations of the outer bar and inner bar are illustrated in figures 4A to 4F.

The orbits of both bars are clockwise, as labelled by the arrows. The arrows represent the sense of the orbits (not the sense of the pattern rotations).

Each bar has pattern rotation (precession) which is in the opposite sense to its own orbits. So the pattern rotation (precession) of both bars is anticlockwise.

The figures show the bars in the inertial frame at sequential time intervals.

During each interval, between one diagram and the next, the outer bar experiences pattern rotation (precession) of 31.6 degrees anticlockwise, and the inner bar experiences pattern rotation (precession) of 10 degrees anticlockwise.

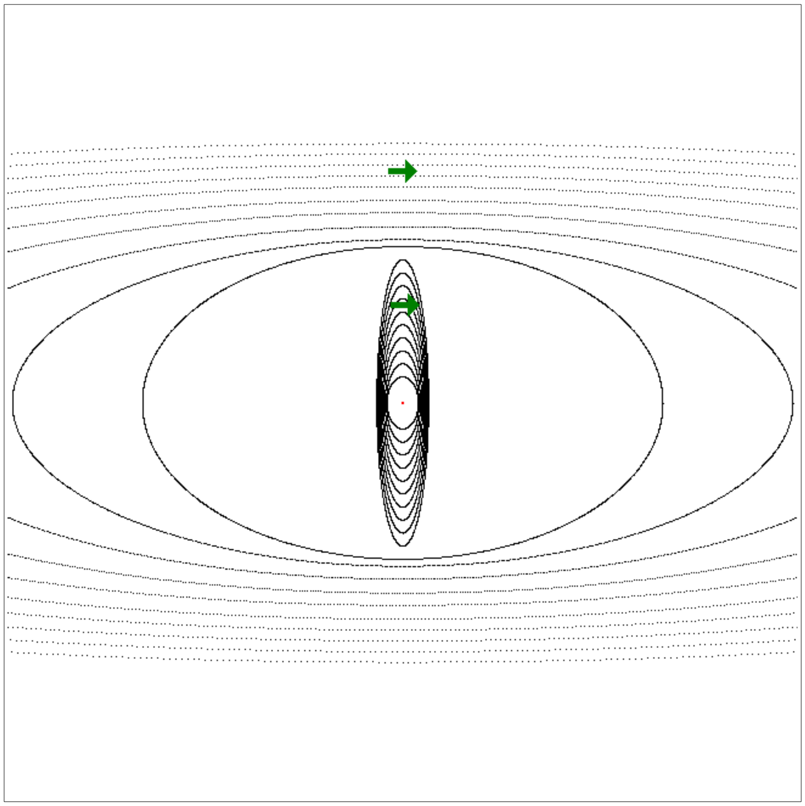


Figure 4 A

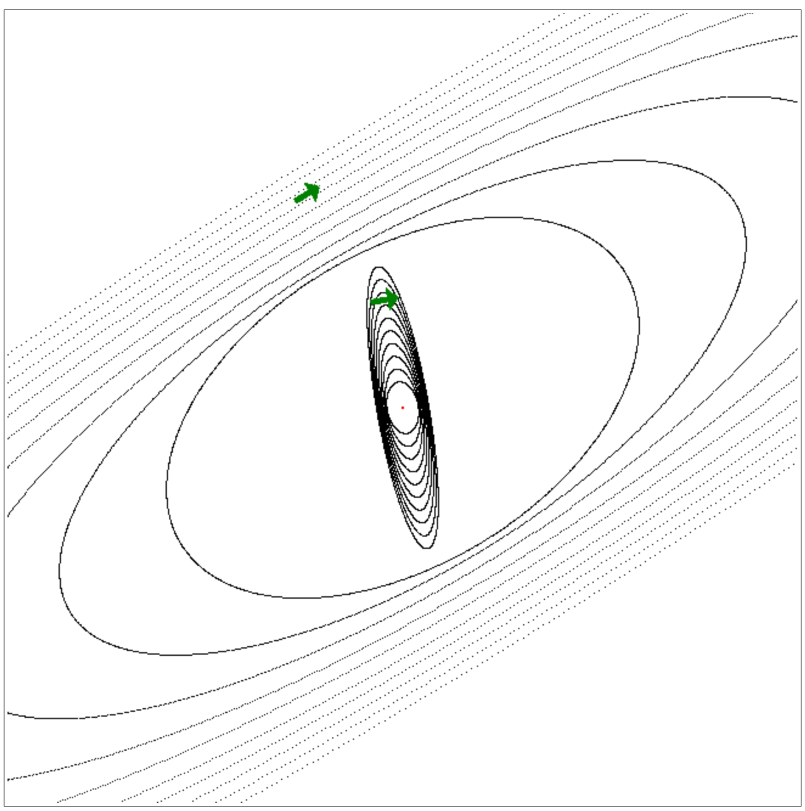


Figure 4 B

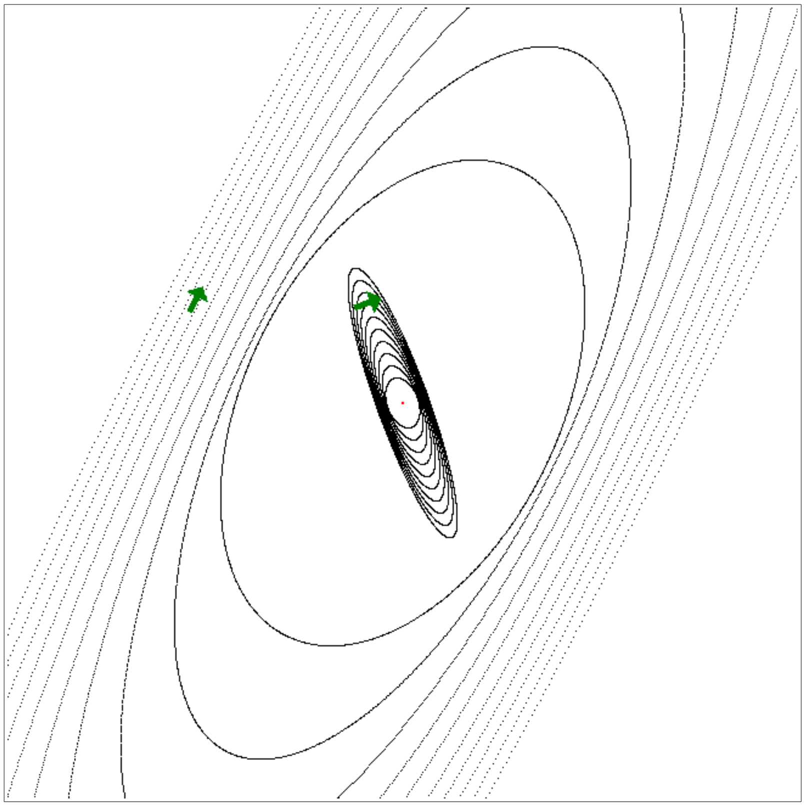


Figure 4 C

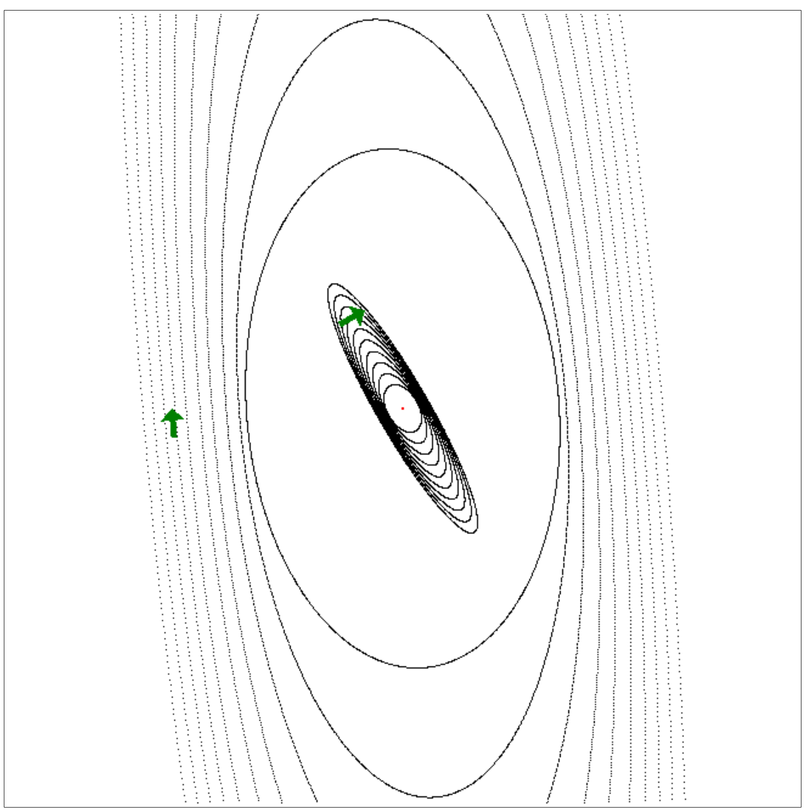


Figure 4 D

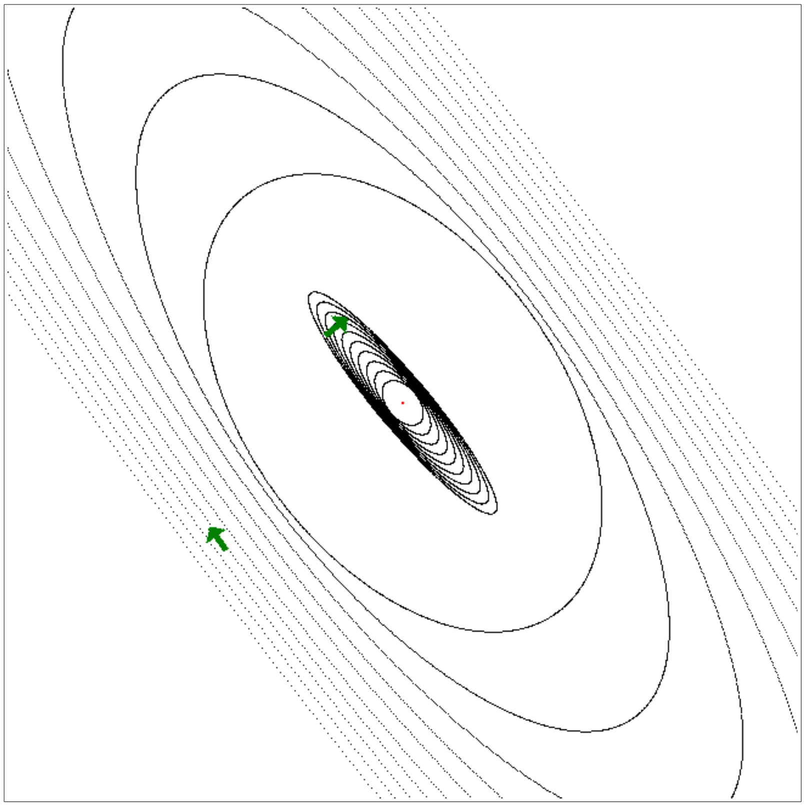


Figure 4 E

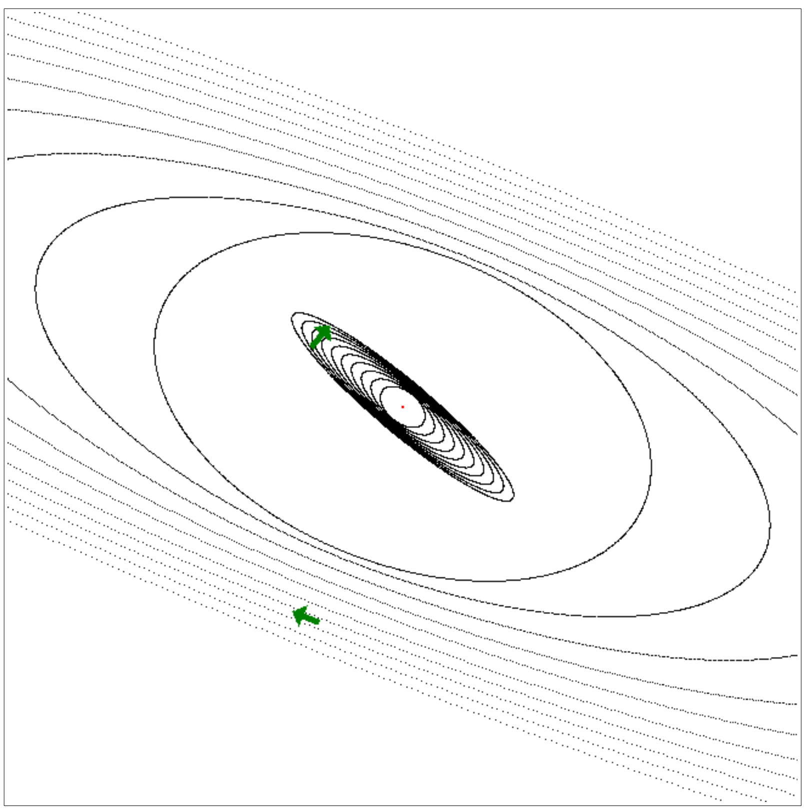


Figure 4 F

Dynamics with orbits of inner bar in opposite sense to orbits of outer bar

The second possibility for the dynamics is with the orbits of the inner bar in the **opposite** sense to the orbits of the outer bar.

The resulting pattern rotations of the outer bar and inner bar are illustrated in figures 5A to 5F.

The orbits of the outer bar are clockwise, and the orbits of the inner bar are anticlockwise, as labelled by the arrows. The arrows represent the sense of the orbits (not the sense of the pattern rotations).

Each bar has pattern rotation (precession) which is in the opposite sense to its own orbits. So the pattern rotation is anticlockwise for the outer bar, but clockwise for the inner bar.

During each interval, between one diagram and the next, the outer bar experiences pattern rotation (precession) of 31.6 degrees anticlockwise, and the inner bar experiences pattern rotation (precession) of 10 degrees clockwise.

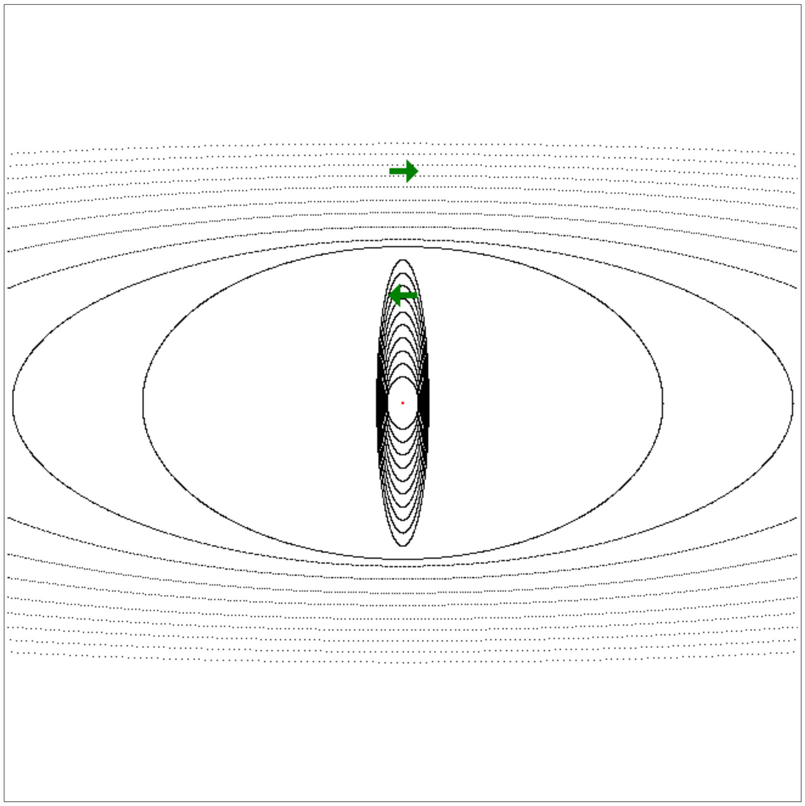


Figure 5 A

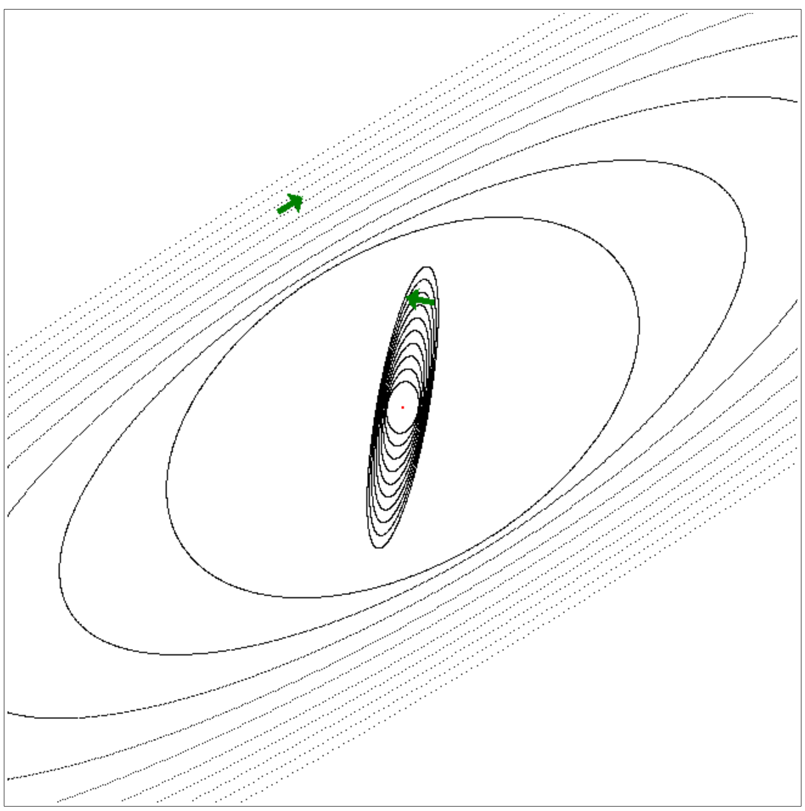


Figure 5 B

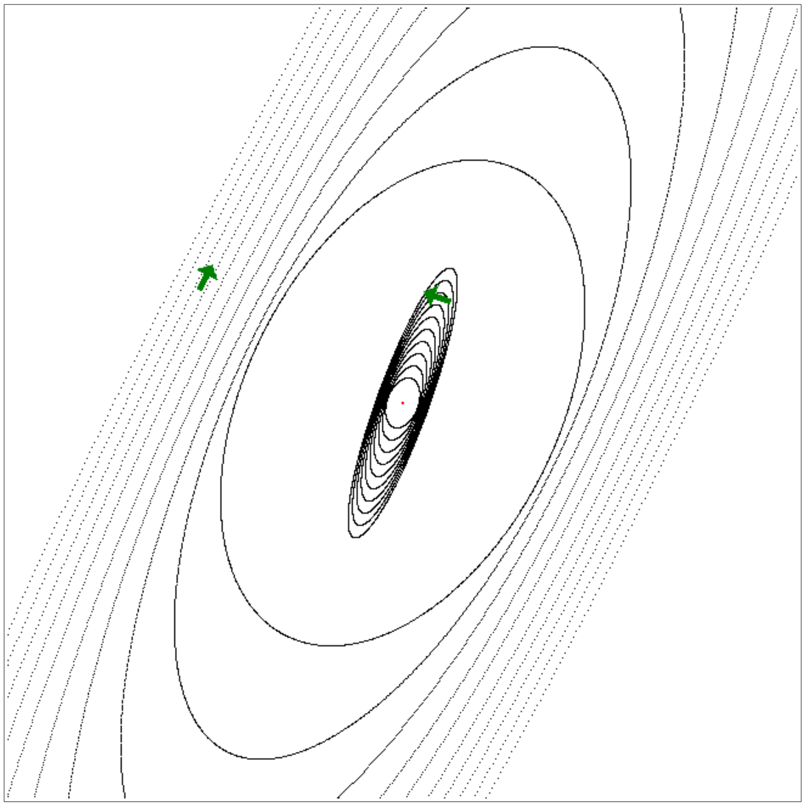


Figure 5 C

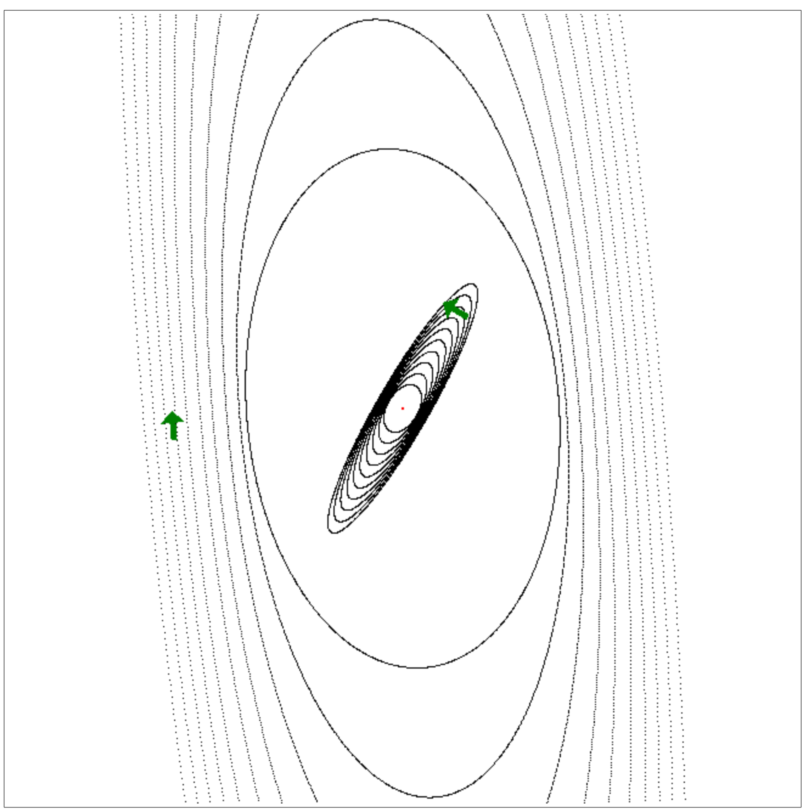


Figure 5 D

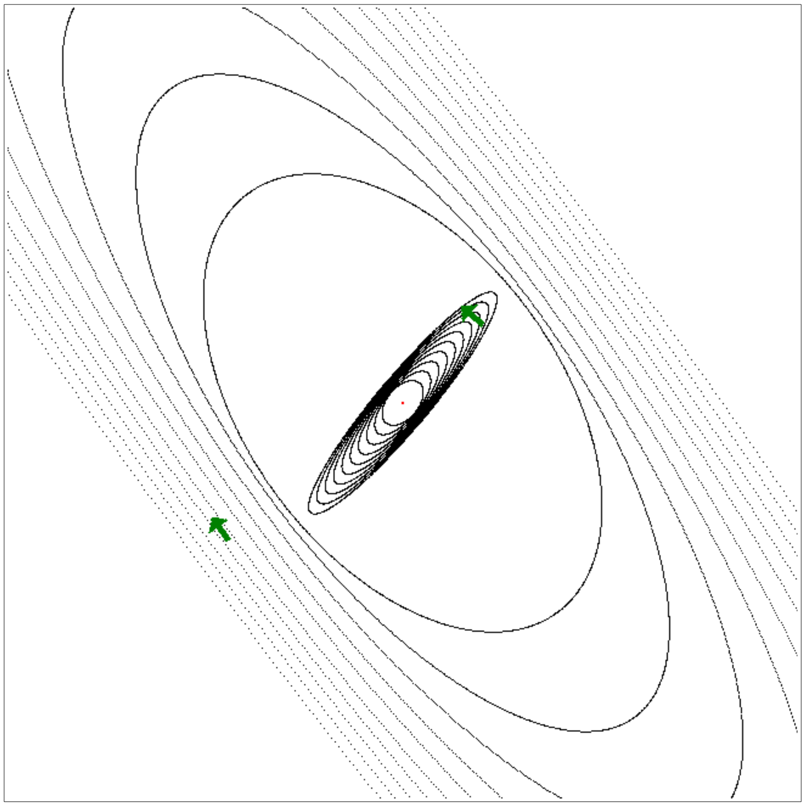


Figure 5 E

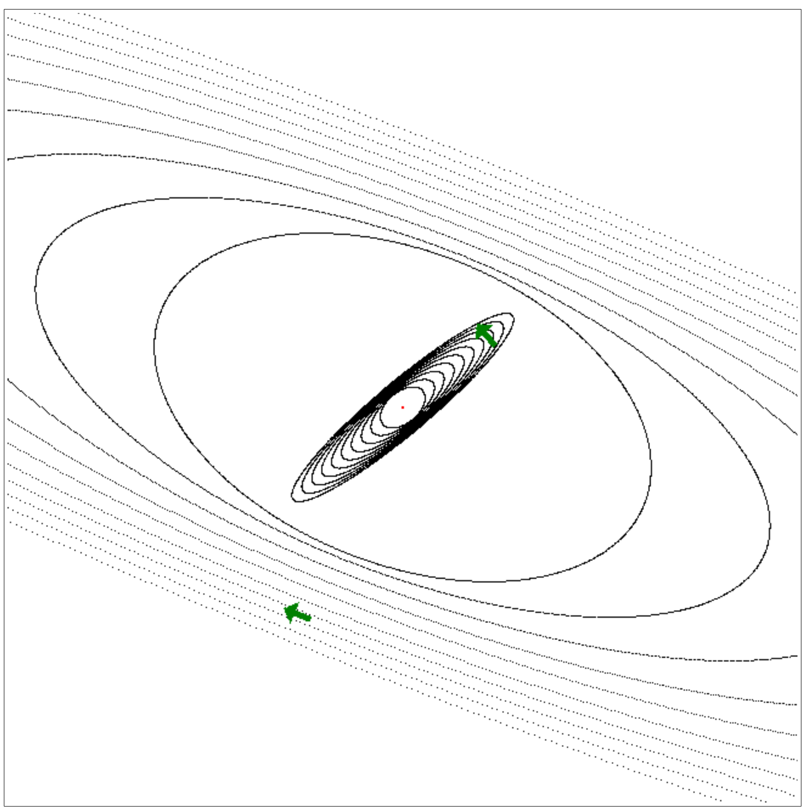


Figure 5 F

Streamline parameters

The starting parameters used by software to produce the orbit streamlines of the nested bars described here, are listed in figure 6, using the system of units described above. The initial position (x, y) and velocity (vx, vy) are listed for one particle of each of 10 example streamlines of the outer bar and 10 example streamlines of the inner bar.
The listed parameters have the orbits of the inner bar in the same sense as the orbits of the outer bar, and so they produce the dynamics illustrated in figures 4A to 4F.

Figure 6: Streamline parameters

x	**y**	**vx**	**vy**
0.2	0	0	0.0485
0.3	0	0	0.0595
0.4	0	0	0.0727
0.5	0	0	0.08696
0.6	0	0	0.1008
0.7	0	0	0.1157
0.8	0	0	0.1298
0.9	0	0	0.1440
1.0	0	0	0.1581
1.1	0	0	0.1730
2	0	0	1.5337
3	0	0	1.8816
4	0	0	2.2990
5	0	0	2.7499
6	0	0	3.1876
7	0	0	3.6588
8	0	0	4.1046
9	0	0	4.5537
10	0	0	5.0000
11	0	0	5.4707

The other possibility, in which the orbits of the inner bar are in the opposite sense to the orbits of the outer bar, is produced by simply multiplying the initial **vy** values of the 10 streamlines of the inner bar by -1, which will produce the dynamics illustrated in figures 5A to 5F.

Generalisation to other simple power-laws in the range x>1

In a separate paper, the dynamics of bars (and nested bars) which have been successfully constructed in the (x=1.5) field will be described and illustrated. The innermost streamline of a bar in that field is much more circular than in the (x=2) field. Nested bars in the (x=1.5) field resemble observed nested bars even better than those illustrated here in the (x=2) field.

Conclusions

This investigation based on simple power-law fields indicates that:

A galactic bar has an inner boundary, beyond which further smaller co-precessing streamlines cannot be added.
Particles existing in the region between that inner boundary, and the galactic centre, and may form an inner bar.
The inner bar has a different pattern rotation rate to the outer bar.
The inner bar is a scaled-down self-similar copy of the outer bar.

The outer bar has pattern rotation in the opposite sense to its own orbits.
The inner bar has pattern rotation in the opposite sense to its own orbits.

There are two possibilities for the dynamics:
A. If the orbits of both bars are clockwise, then the pattern rotations of both bars are anticlockwise.
B. If the orbits of the outer bar are clockwise, and the orbits of the inner bar are anticlockwise, then the pattern rotation of the outer bar is anticlockwise, and the pattern rotation of the inner bar is clockwise.

The inner bar has a slower absolute pattern rotation rate than the outer bar.

The ratio of the absolute pattern rotation rates of the outer and inner bars can be calculated from the power-law exponent and the scale ratio.

Discussion

This investigation uses computer modelling based on the assumption that every orbital streamline of a bar interacts with its neighbour streamlines, and that the effect of the interactions is to modify the ellipticity of each streamline, producing a precisely-tuned ellipticity profile, so that every streamline of the bar has exactly the same precession rate.
There is a clear and strong precedent for this remarkable mechanism, and that is in the dynamics of the eccentric rings of Uranus and Saturn [2]. The very existence of these eccentric rings is the beautiful result of the streamlines interacting to produce exactly the precise eccentricity profile which makes the precession rate of every streamline equal. [3] [4]

This study, based on simple power-law fields in the range $x>1$, indicates that the inner bar will have a slower absolute pattern rotation rate than the outer bar. Generally the literature presents the inner bar as having an absolute pattern rotation rate which is equal to or greater than that of the outer bar. However, as pointed out in [5], systems with inner bar rotating slower than the outer bar have never been excluded on theoretical grounds.
The possibility that an inner bar may have a slower absolute pattern rotation rate than the main bar was proposed by [6] based on modelling of nested bars, which produced in some models a nuclear gaseous bar persisting for a limited time with pattern rotation 2 to 3 times slower than that of the main bar.

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