

# Theoretical Orbits of Planets in Binary Star Systems

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# 1: Introduction

A binary star system consists of two stars which orbit around their joint centre of mass. A large proportion of stars belong to such systems. What sorts of orbits can planets have in a binary star system?

To examine this question we use a computer program called a multi-body gravitational simulator. This enables us to create accurate simulations of binary star systems with planets, and to analyse how planets would really behave in this complex environment.

Initially we examine the simplest type of binary star system, which satisfies these conditions:-

1. The two stars are of equal mass.
2. The two stars share a common circular orbit.
3. Planets orbit on the same plane as the stars.
4. Planets are of negligible mass.
5. There are no tidal effects.

We use the following units:-

One time unit = the orbital period of the star system.

One distance unit = the distance between the two stars.

We can classify possible planetary orbits into two types. A planet may have an internal orbit, which means that it orbits around just one of the two stars. Alternatively, a planet may have an external orbit, which means that its orbit takes it around both stars.

Also a planet's orbit may be prograde (in the same direction as the stars' orbits), or retrograde (in the opposite direction to the stars' orbits).

## 2: Large External Orbits

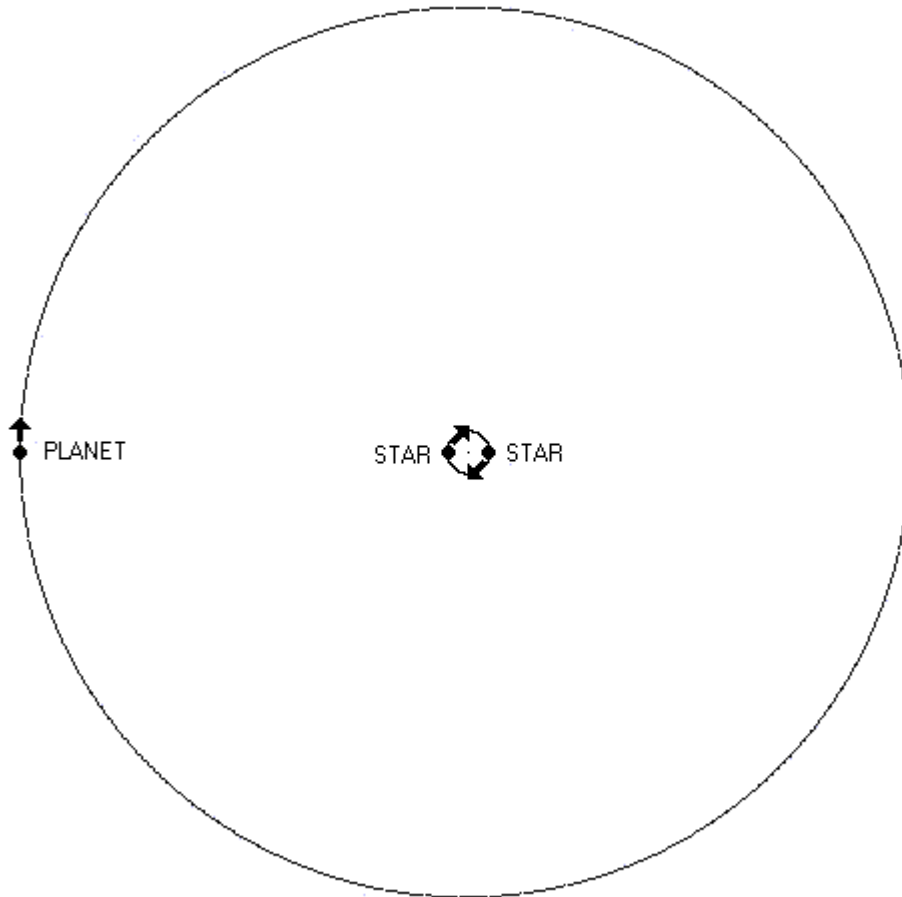
In table 1 the gravitational field of a single star is compared with the external gravitational field of a simple binary system with the same total mass. The distance between the two binary stars = 1 unit, so each star is 0.5 unit from the system centre. For clarity we use a unit of acceleration such that at distance = 8 the acceleration exerted by the single star = 1 unit.

distance from system centre	single star field strength	binary field strength (average)	binary field strength (minimum)	binary field strength (maximum)
8	1.000	1.003	0.944	1.012
7	1.306	1.311	1.296	1.326
6	1.777	1.787	1.759	1.815
5	2.560	2.579	2.522	2.638
4	4.000	4.048	3.910	4.192
3	7.111	7.263	6.825	7.732
2	16.00	16.80	14.61	19.34
1	64.00	79.72	45.79	142.2

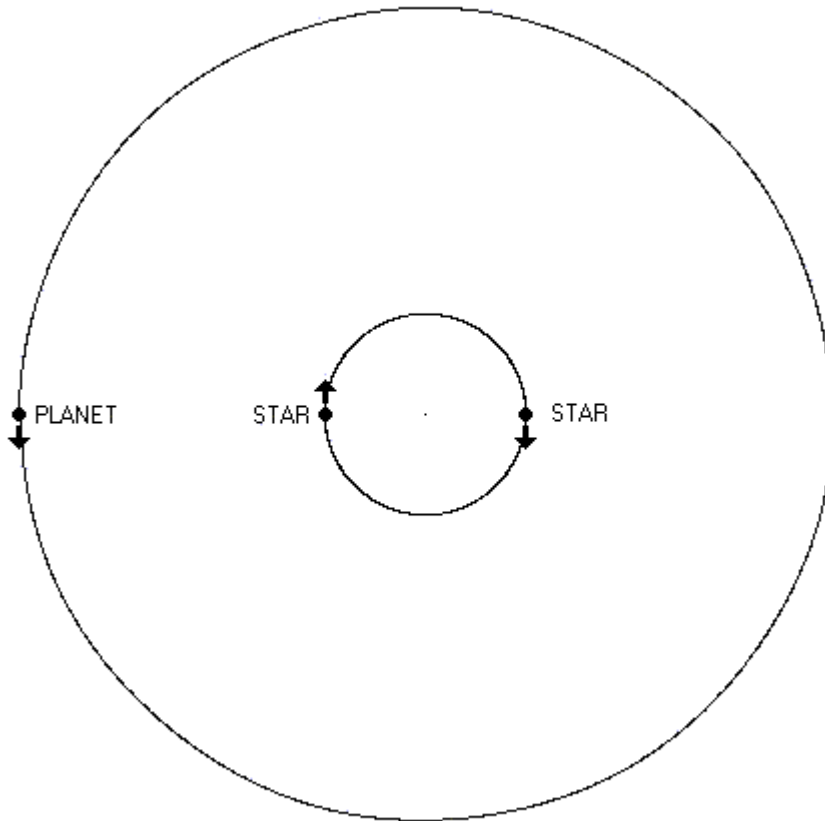
The field strength of the single star at any given point is constant. However, the field strength of the binary system at any given point fluctuates cyclically over time. The period of the fluctuation in a simple binary (as defined above) = 0.5 (i.e. half the period of the star system). The magnitude of the fluctuation is relatively very small at distance = 8, but increases as we get closer to the star system. At distance = 1 the fluctuation is very large indeed.

At distance = 8, the average field strength of the binary system (averaged over a complete period of the binary system) is similar to the field strength of the single star. However, as we get closer to the stars, the binary average field strength increases more rapidly than the single star field strength. The gravitational field of the single star obeys the inverse square law. The average gravitational field of the binary system is the combination of the individual inverse-square fields of the two moving stars, but it is important to note that this combined field does not follow the inverse-square rule.

In this diagram a planet has an external orbit at distance  $d = 10$  from the star system centre. At this relatively large distance, the combined gravitational field of stars A and B is similar to the field of a single star with their combined mass. Therefore the planet's orbit is very similar to an ellipse. (In this case we have shown an orbit with eccentricity = 0, so the orbit is circular). At large distances like this, retrograde and prograde orbits are both viable. We've shown a prograde orbit, The planet's orbital period  $p = 31.6$ .



In the next diagram, a planet has a retrograde orbit around the binary star system at distance  $d = 2$ . The fluctuating gravitational field causes deviations from a circular path but the orbit is perfectly stable. The period  $p = 2.77$



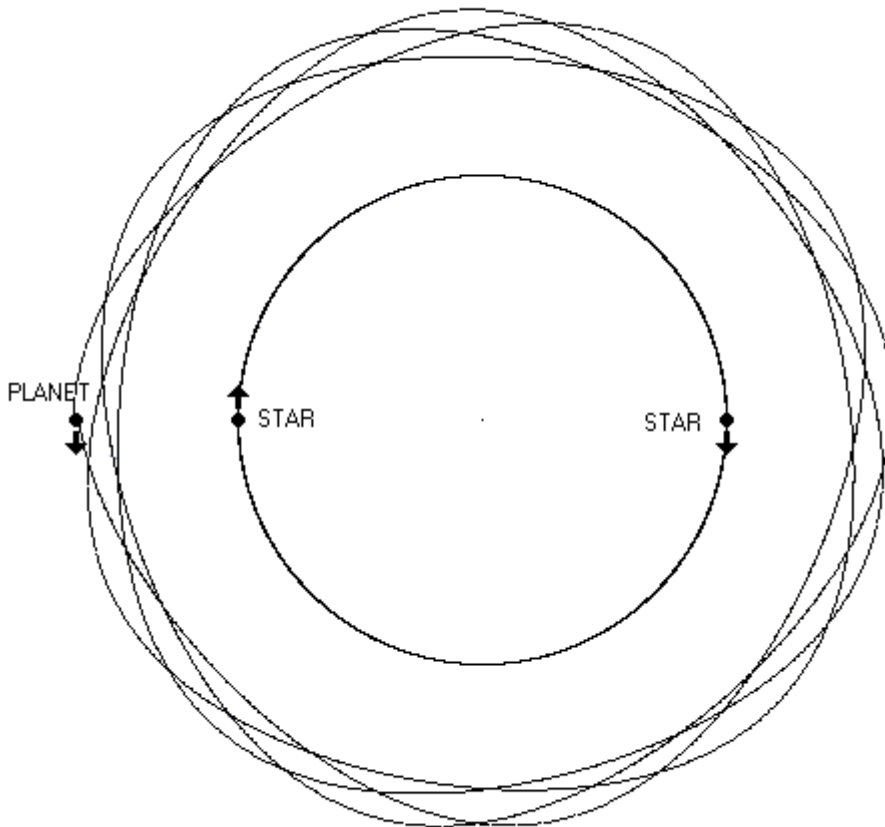
Surprisingly, a prograde orbit at the same distance  $d = 2$  would be unstable. Why is the retrograde orbit stable and the otherwise identical prograde orbit unstable?

It is because the gravitational field strength fluctuations experienced by the prograde planet have the same amplitude but a longer period than the fluctuations experienced by the retrograde planet. The longer period of fluctuation has a destabilizing effect.

In fact, at distances of less than  $d = 2$ , prograde orbits become even more unstable. The following examples of external orbits are possible only in a retrograde direction.

### 3: Small External Orbits

In the next diagram, a planet has a retrograde orbit with a period  $p = 0.6$ . It is interesting that the planet completes an orbit in less time than the two stars take to complete their orbits. The planet's distance from the star system centre varies between  $d = 0.84$  and  $d = 0.74$ .

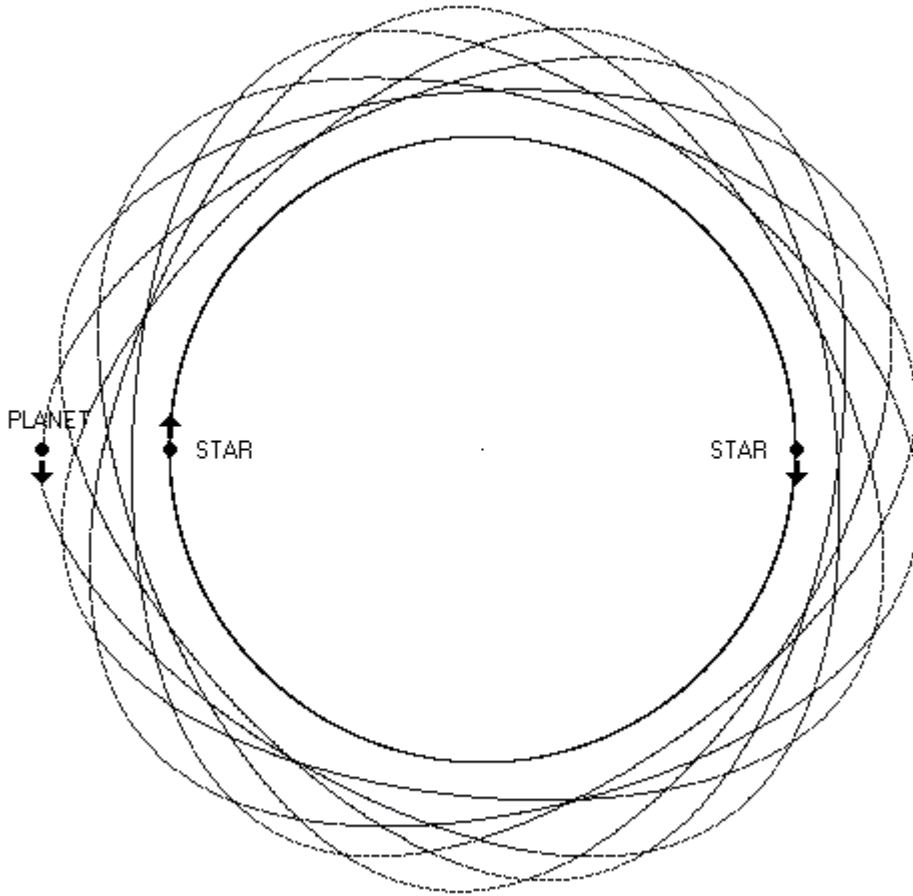


The typical shape of the quasi-circular orbit is easy to see here. It comprises regular fluctuations to either side of a truly circular path. These fluctuations are synchronous with the alignment of the planet and the two stars.

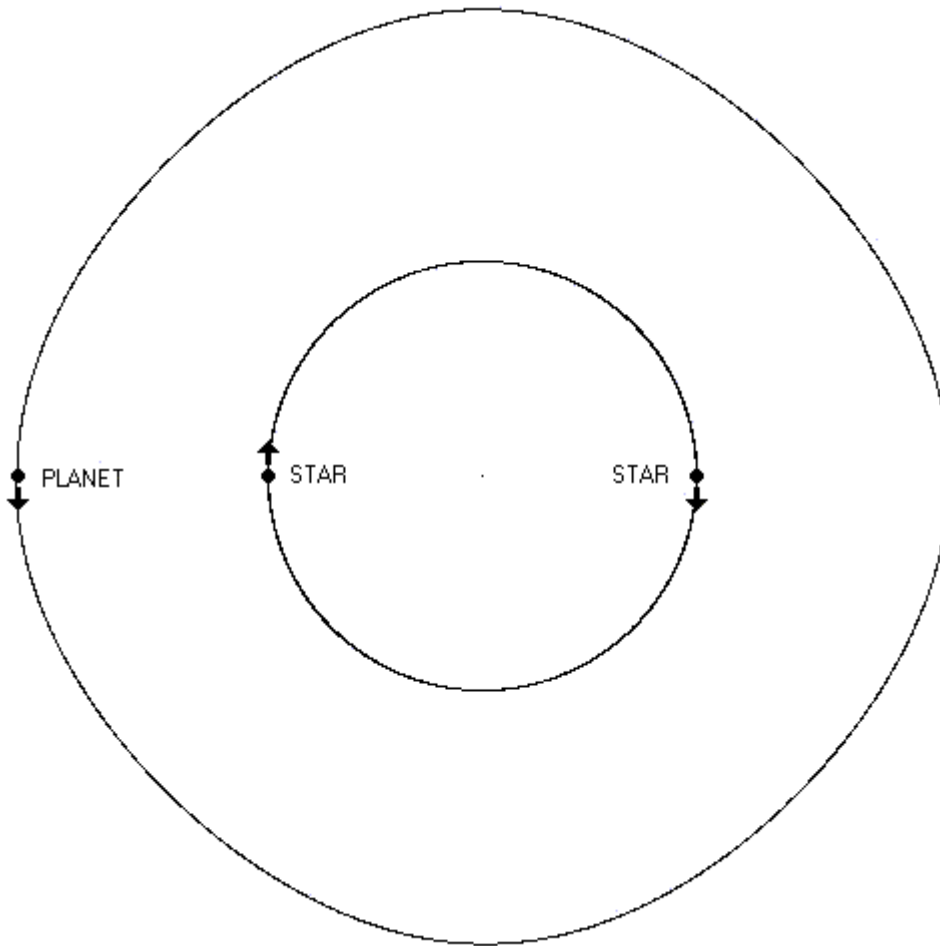
When the planet and both stars are in conjunction, i.e. lined up in a straight line, the planet is at its maximum distance from the star system's centre.

When the planet is equidistant from both stars, the planet is at its minimum distance from the star system's centre.

In the next diagram, a planet has a retrograde orbit with a period  $p = 0.42$ . The planet's distance from the star system centre varies between  $d = 0.70$  and  $d = 0.55$ . The variation in distance is much greater than in the previous example, because the gravitational field of the star system fluctuates more at this proximity.

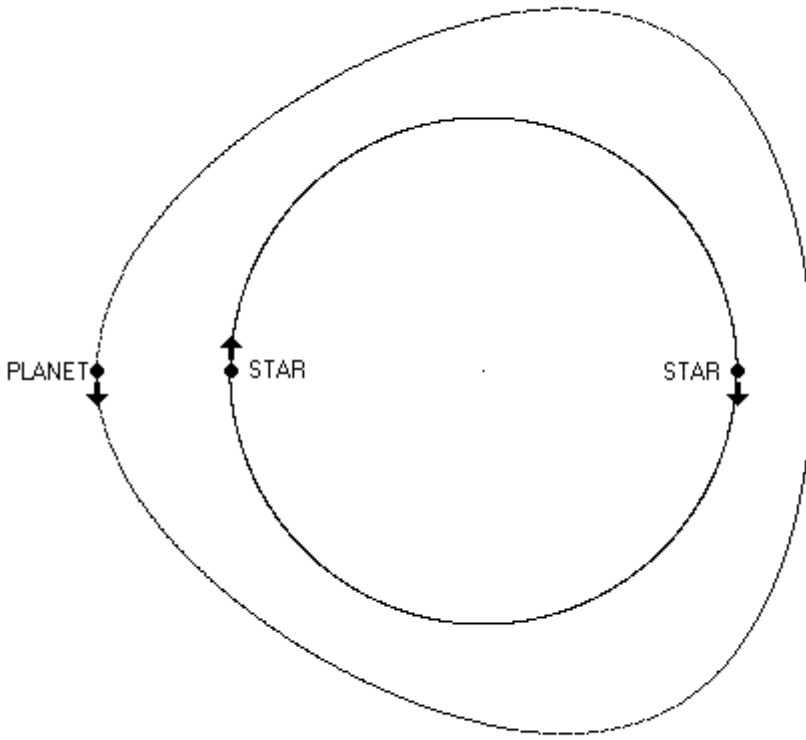


In the next diagram, a planet has a retrograde orbit with a period  $p = 1$ . The orbit shape is a rounded square because the planet experiences exactly 4 conjunctions per orbit.

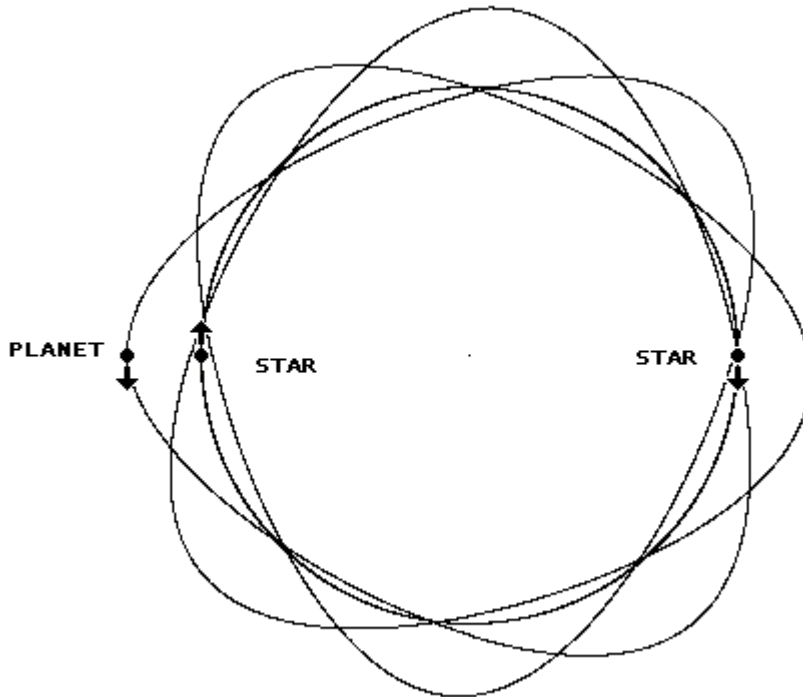




In the next diagram, a planet has a quasi-circular orbit with a period  $p = 0.5$ . The orbit shape is a rounded triangle because the planet experiences exactly 3 conjunctions per orbit.



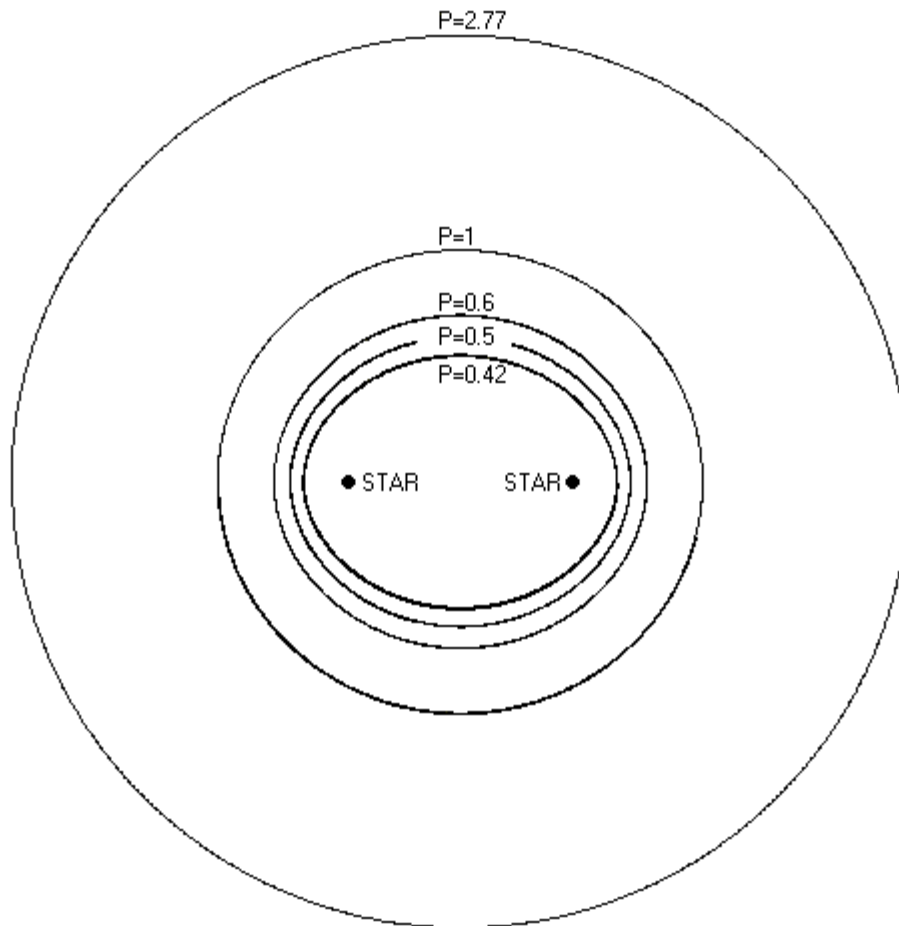
In the next diagram, a planet has a quasi-circular orbit with a period  $p = 0.33$ . Note that even though the planet has an external orbit, it sometimes comes closer to the centre of the star system than does either star.



The minimum possible period for a planet with an external quasi-circular orbit will be determined by the actual physical sizes of the stars and planet. As we simulate orbits which take the planet closer and closer to the stars, eventually instability will occur because of tidal effects.

In the next diagram we re-examine the shape of some of the above external quasi-circular orbits, using a viewing frame which rotates with the star system, so that the stars appear to not move.

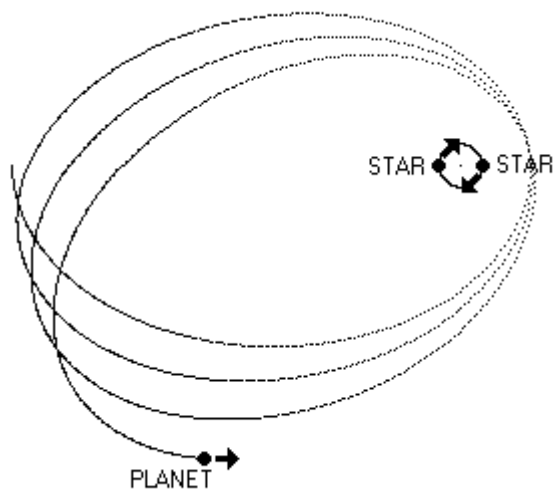
The numbers in the diagram state the period of each planet's orbit (relative to the period of the star system).



## 4: Eccentric External Orbits

In every example so far we have adjusted the planet's trajectory to make its orbit as close to circular as possible. But what happens if the planet's orbit is eccentric?

The next diagram shows a planet with an eccentric retrograde orbit around a binary star system. The orbit is approximately elliptical. The planet's period  $p = \text{approx. } 12.5$  and its maximum distance = 10. The shape of the orbit slowly rotates around the star system in the same direction as the planet's orbital motion. This is called apsidal advance, and occurs here because the gravitational field of the binary does not obey the inverse square law, as shown in table 1.



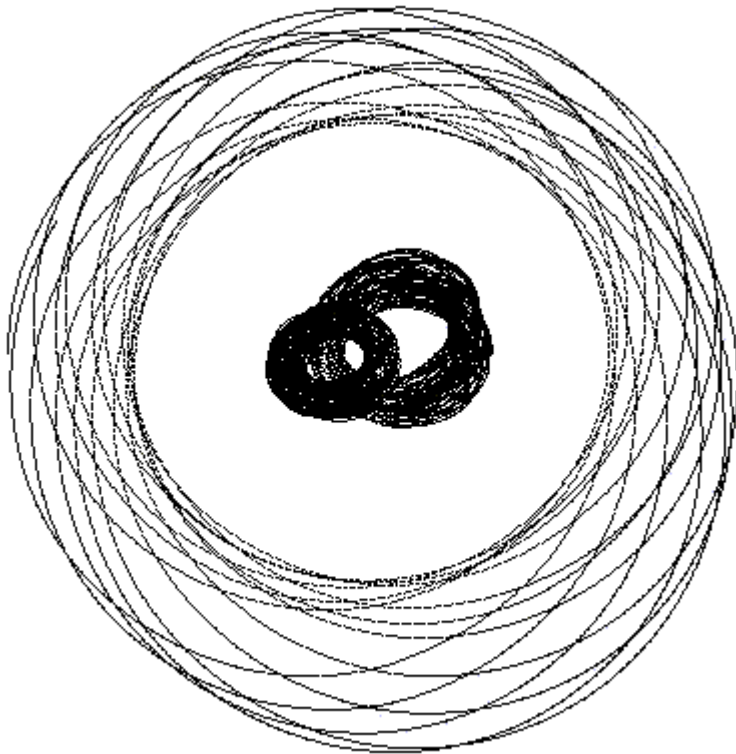
## 5: Complex External Orbits

So far we have looked at planets orbiting a simple binary star system (which satisfies the conditions defined near the beginning of this article). Can planets survive in more complex binary star systems?

In the next diagram we look at just one rather extreme example. It's a binary star system in which the stars have unequal masses, and eccentric orbits around each other. The planet has an eccentric retrograde external orbit. Moreover the planet has a significant mass. The ratio of masses of the planet and the two stars is 1:10:20. This orbit is more complicated than the previous examples, because the gravitational field is more complicated. However, the orbit is stable.

The planet perturbs the orbits of the stars with several effects. The centre of mass of the binary star pair orbits around the centre of mass of the entire trinary system. The orbital motions of the two stars around each other are not exactly elliptical. And the orbits of the stars experience apsidal advance.

The smallest ellipse is the orbit of the more massive star. The medium-size ellipse is the orbit of the less massive star. The large ellipse is the orbit of the planet.

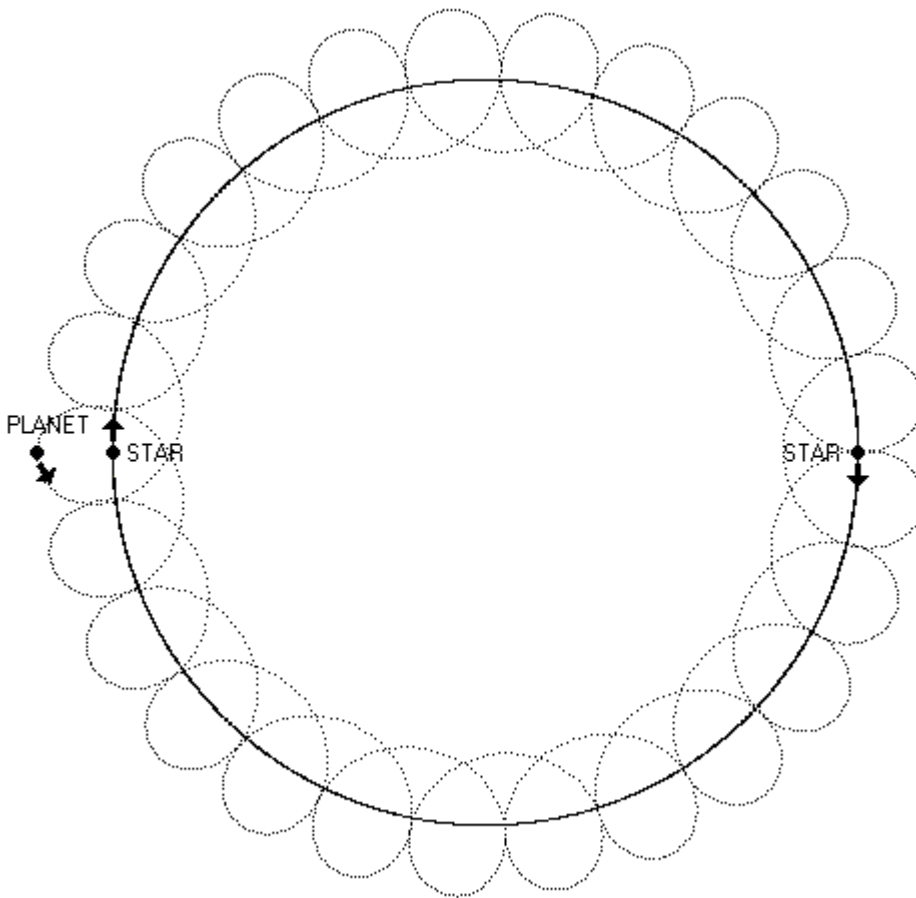


Orbits in which a planet passes close enough to a star for tidal effects to be large will tend to be unstable.

Inclination of a planet's orbit relative to the orbits of the stars around each other would introduce a further complexity in a planet's orbit. This is beyond the scope of this article, except to say that this extra complexity is not necessarily destabilizing.

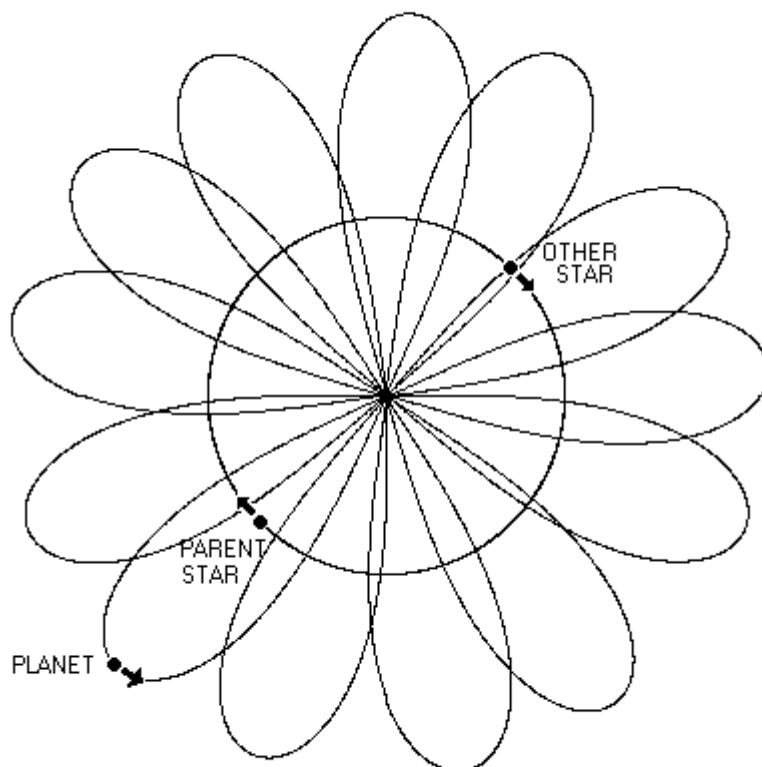
## 6: Internal orbits

In the next diagram, the planet has an orbit around just one of the stars (its parent star). The distance between the planet and its parent star is very small relative to the distance between the two stars. At this relative proximity the gravitational field affecting the planet is almost the same as it would be if its parent star were a single star. So the planet has a simple elliptical orbit (circular in this example). The orbital period  $p = 0.042$ . We've shown a retrograde orbit. At relatively small distances like this, retrograde and prograde orbits are both viable.

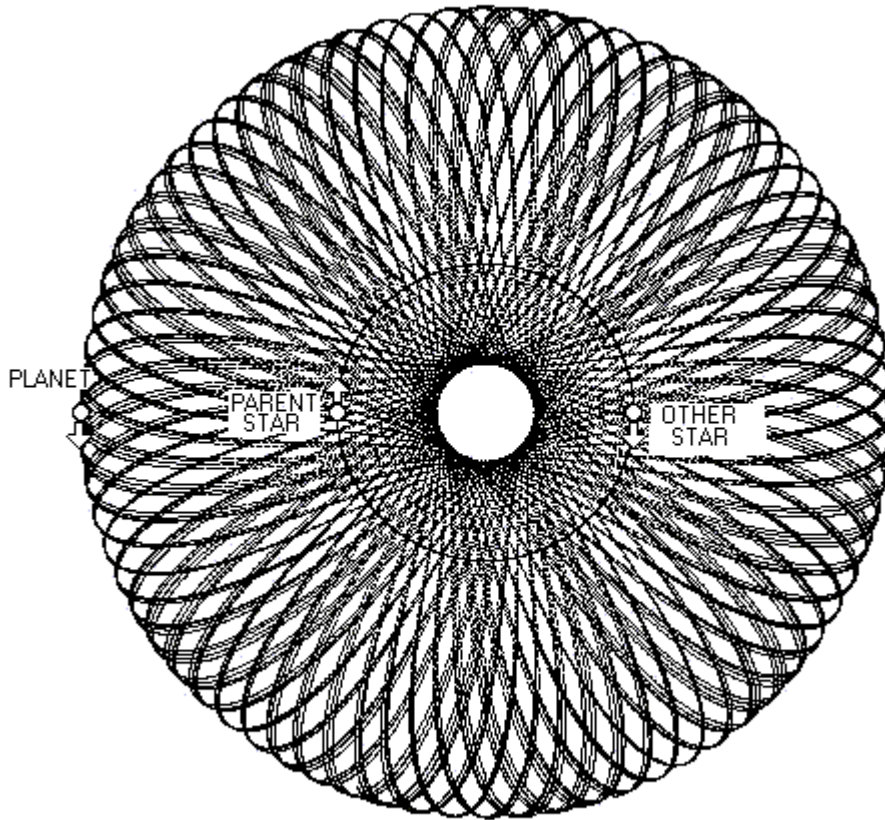


However, as we examine relatively larger internal planet orbits, prograde orbits become increasingly unstable. The following orbits, which are stable in the retrograde direction, would all be impossible in the prograde direction.

In the next diagram, the planet has a retrograde internal orbit at a distance of approx  $d = 0.5$  from its parent star.

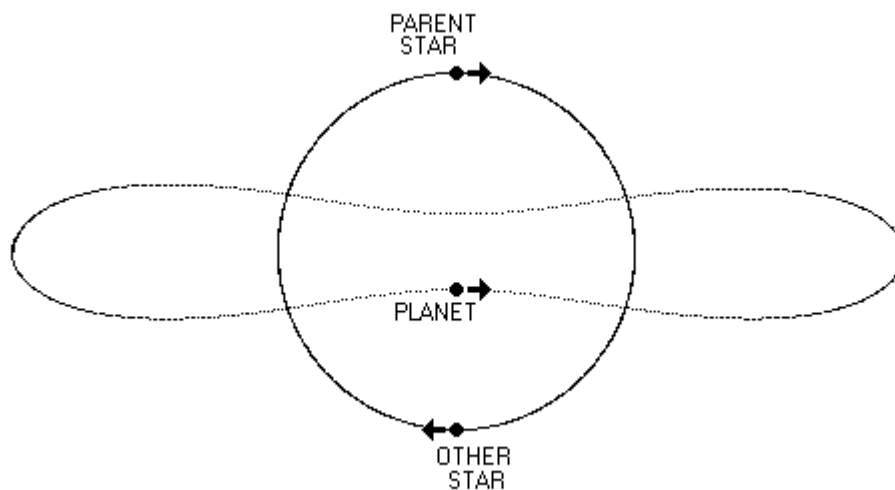
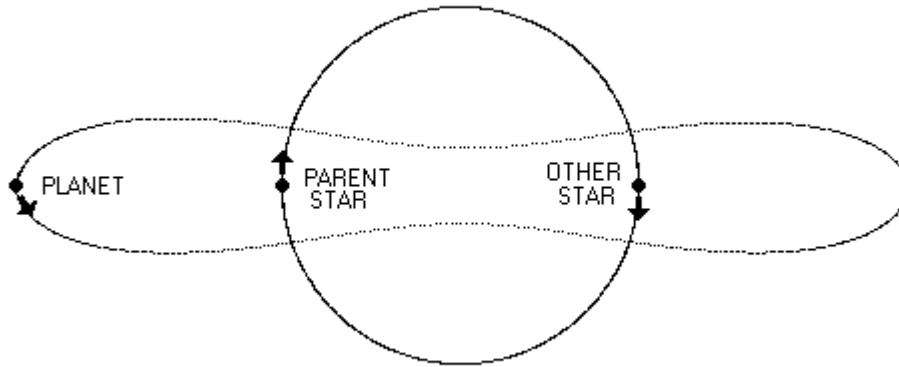


In the next diagram, the planet has an internal orbit at a distance of approx  $d = 0.67$  from its parent star.



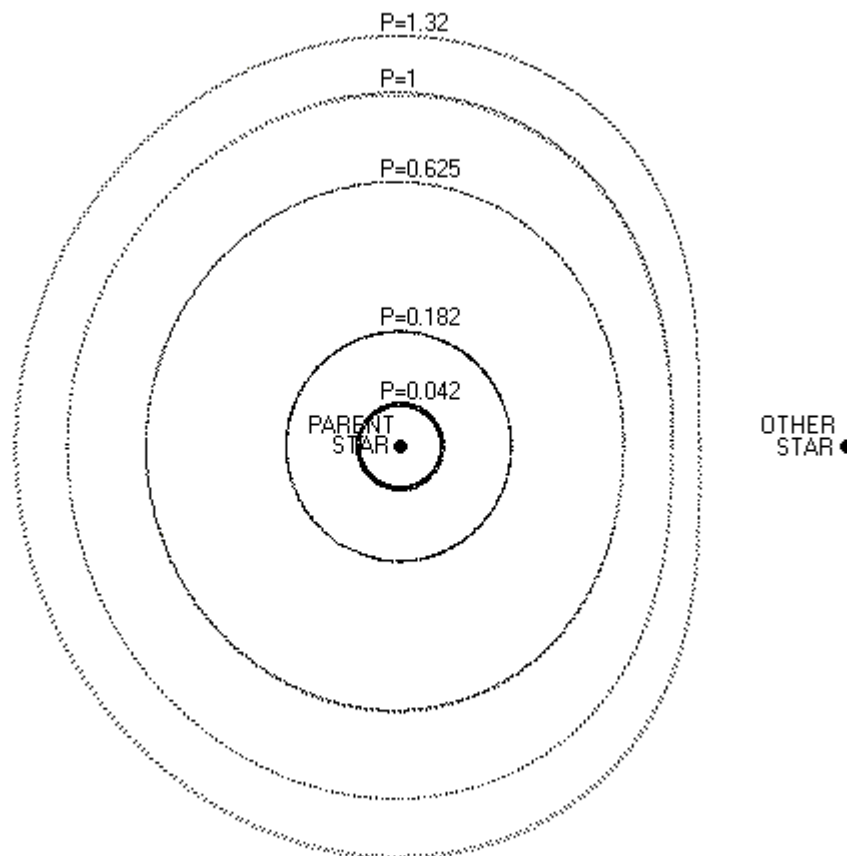


The next two diagrams show a planet orbiting its parent star with a period  $p = 1$ . The orbit is quasi-circular around star A, but star A is moving in its own orbit too. The combination of these two motions with exactly the same period results in an ice-cream-spoon shaped orbit for the planet. As shown in the second diagram, the planet sometimes approaches the other star closer than it ever gets to its parent star.



In the next diagram we examine the shape of internal quasi-circular orbits by using a viewing frame which rotates with the star system, so that the stars appear to not move.

The numbers in the diagram state the period of each planet's orbit (relative to the period of the star system).



If we relax any of the conditions specified in the introduction, even more complex internal orbits arise.

## 7: Conclusion

In general, most binary star systems are capable of having planets.

Most binary systems are capable of having planets in external orbits (orbiting around both stars) with relatively large orbit radii. In some favorable systems, external orbits with relatively small orbit radii are possible, with periods equal to or even less than the star system period.

Additionally, many binary systems are capable of having planets with internal orbits (orbiting around just one of the stars) of relatively small orbit radii, providing that the separation between the two stars is always much greater than the diameters of the stars. In some favorable systems, internal orbits with relatively large orbit radii are possible, with periods equal to or even longer than the star system period.

Indeed, some binary systems are capable of supporting planets orbiting star A, other planets orbiting star B, and further planets orbiting both stars.

The range of stable orbits is much greater for retrograde orbits than than prograde orbits.

The presence of a planet with significant mass in a binary star system may cause wobbling of the binary, or of one of its stars. Therefore the same methods which have successfully detected planets orbiting single stars can be used to search for planets in binary star systems. Some binary systems may offer further clues which might indicate the possible presence of one or more massive planets.

1. Apical advance of eccentric binaries.
2. Variations in the radial velocity curves of spectroscopic binaries.
3. Variations in the eclipse cycles of eclipsing binaries.