Trailing or leading spiral arms

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Summary

Single-armed and two-armed spiral density waves are examined theoretically in four ranges of simple power-law fields, and the trailing or leading senses of the arms are determined.

Method

The results are obtained from a combination of:
(1) The orbital characteristics and scale-free properties of simple power-law fields.
(2) The relationship between ellipticity and apsidal angle in simple power-law fields.
(3) Modelling using software which measures precession rates of elliptical orbits in any chosen gravitational field.

Definitions

The fields examined are simple power-law fields in which the acceleration towards the centre is proportional to the distance from the centre raised to some power x. Each field is described its value of x.

The apsidal angle of an orbit is defined as the angle from one apocentre to the next apocentre.

A streamline is defined as a large number of particles whose orbits are all of the same size and ellipticity, and configured so that, when viewed in a frame which co-rotates at the precession rate of the orbits, all the orbits form one ellipse.
An m=1 streamline is defined as a streamline which, when viewed in a frame which co-rotates at the streamline's precession rate, is an ellipse with one of its foci at the centre of attraction.
An m=2 streamline is defined as a streamline which, when viewed in a frame which co-rotates at the streamline's precession rate, is an ellipse with its centre at the centre of attraction.

It is important to note that the definition of precession for m=1 streamlines is different to the definition of precession for m=2 streamlines.

For streamlines of mode m=1, the apsidal angle which gives zero precession is $\theta=2\pi$.
The (x=-2) field (keplerian field) has apsidal angle $\theta=2\pi$ for all orbits.
Therefore m=1 streamlines have zero precession in the (x=-2) field,
and prograde precession in fields (x<-2),
and retrograde precession in fields (x>-2).

For streamlines of mode m=2, the apsidal angle which gives zero precession is $\theta=\pi$.
The (x=1) field (harmonic field) has apsidal angle $\theta=\pi$ for all orbits.
Therefore m=2 streamlines have zero precession in the (x=1) field,
and prograde precession in fields (x<1),
and retrograde precession in fields (x>1).

Results

A. For fields in the range (-2.36<x<-2):
Streamline mode m=1
Precession of the m=1 streamlines is prograde relative to the orbits.
If a set of nested co-precessing and non-colliding m=1 streamlines forms, it will have:
- Streamline ellipticity increasing with increasing streamline size.
- Pattern rotation in the same sense as the orbits.
- If a spiral density wave pattern forms, it will be:
  - **single-armed**, **trailing** relative to the orbits, and trailing relative to the pattern rotation.

B. For fields in the range (-2<x<-1.222):
Streamline mode m=1
Precession of the m=1 streamlines is retrograde relative to the orbits.
If a set of nested co-precessing and non-colliding m=1 streamlines forms, it will have:
- Streamline ellipticity increasing with increasing streamline size.
- Pattern rotation in the opposite sense to the orbits.
- If a spiral density wave pattern forms, it will be:
  - **single-armed**, **leading** relative to the orbits, but trailing relative to the pattern rotation.

C. For fields in the range (-1.222<x<1):
Streamline mode m=2
Precession of the m=2 streamlines is prograde relative to the orbits.
If a set of nested co-precessing and non-colliding m=2 streamlines forms, it will have:
- Streamline ellipticity decreasing with increasing streamline size.
- Pattern rotation in the same sense as the orbits.
- If a spiral density wave pattern forms, it will be:
  - **two-armed**, **trailing** relative to the orbits, and trailing relative to the pattern rotation.

D. For fields in the range (1<x<13):
Streamline mode m=2
Precession of the m=2 streamlines is retrograde relative to the orbits.
If a set of nested co-precessing and non-colliding m=2 streamlines forms, it will have:
- Streamline ellipticity increasing with increasing streamline size.
- Pattern rotation in the opposite sense to the orbits.
- If a spiral density wave pattern forms, it will be:
  - **two-armed**, **trailing** relative to the orbits, but leading relative to the pattern rotation.

Future expansion of this work

In an expanded version of this paper it is planned to identify specific examples of each of these four types of spiral arms, from the real structures which we actually observe: galactic grand spiral disks, galactic bars, circum-nuclear galactic disks, disks around single stars, disks around one of a pair of binary stars, circum-binary disks, and eccentric rings around planets.

Conclusion

In this examination of simple power-law fields, it is found that:

Two-armed spiral density waves are of two distinct types,
Both types are **always trailing** relative to the orbits.

Single-armed spiral density waves are of two distinct types, one trailing, and one leading.
If the field is in the range ($x<-2$) then the single arm is **trailing** relative to the orbits.
If the field is in the range ($x>-2$) then the single arm is **leading** relative to the orbits.

References

(To be added)