A theoretical galactic disk in a power-law central field $f \propto r^{0.75}$

The power-law central field examined here is defined by $f \propto r^{0.75}$ where r is the distance from the center of the galaxy and f is the gravitational force towards the center of the galaxy.

A hypothetical galactic disk is constructed, made from concentric orbital streamlines, all centred on the galactic centre. Each streamline is assumed to be comprised of a very large quantity of particles. The behaviour of each streamline is measured by numerical integration software, purely by applying the power-law central force. A scale-free dimensionless system of units for mass and distance and time is used, in which the gravitational constant G=1.

At first all the streamlines are given an identical axis ratio of 0.4826. This causes every streamline to precess at a slightly different precession rate (as illustrated by the red line in Figure 3). The different precession rates will gradually cause each streamline to intersect with or collide with its inner and outer neighbours, making the long-term existence of the disk impractical. In other words, the disk has a winding problem.

In most power-law central fields the angle between successive apocentres, and thus the apsidal precession rate, is a function of the streamline's axis ratio, see references [1] [2] [3] [4]. For a centered elliptical streamline with a given apocentre distance in the central power-law field $f \propto$ r^0.75, increasing the axis ratio of the streamline (making the streamline more circular) will increase its precession rate, whereas decreasing the axis ratio (making the streamline more elliptical) will decrease its precession rate.

It follows that by carefully adjusting the axis ratios of the orbital streamlines, it may be possible to adjust all their precession rates. The aim here is to make all the streamlines precess at one single common rate and furthermore to ensure that they are non-intersecting. Therefore the axis ratio of each streamline is carefully tuned (by slightly adjusting its speed at apocentre) until its precession rate equals the precession rate of the reference streamline.

Starting with a reference streamline at apocentre distance = 1, its precession rate is measured. Next the axis ratios of its inner and outer neighbours are carefully adjusted to achieve that same precession rate. This continues by making a similar adjustment to every streamline. The result is that every streamline is now slightly more circular than its inner neighbour, and slightly less circular than its outer neighbour. In other words, the smaller streamlines are more elliptical, and the larger streamlines are less elliptical. This is a carefully tuned negative ellipticity gradient. The resulting set of streamlines is illustrated in Figure 1.

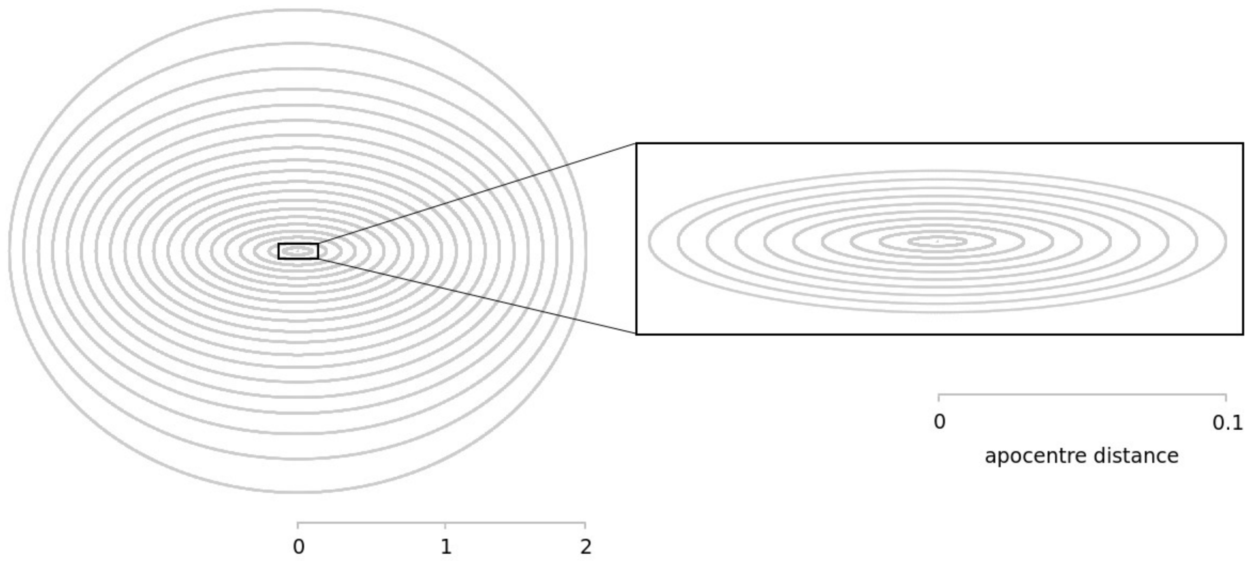

Figure 1: tuned orbital streamlines in field f ∝ r^0.75

The right pane of Figure 1 is a magnification of the central area of the left pane. The left pane shows the larger streamlines with apocentre distances from 0.1 to 2. The right pane shows the smaller streamlines with apocentre distances from 0.01 to 0.1.

The axis ratio is defined as the streamline’s minor axis divided by its major axis. The tuned axis ratios of the streamlines are graphed in Figure 2.

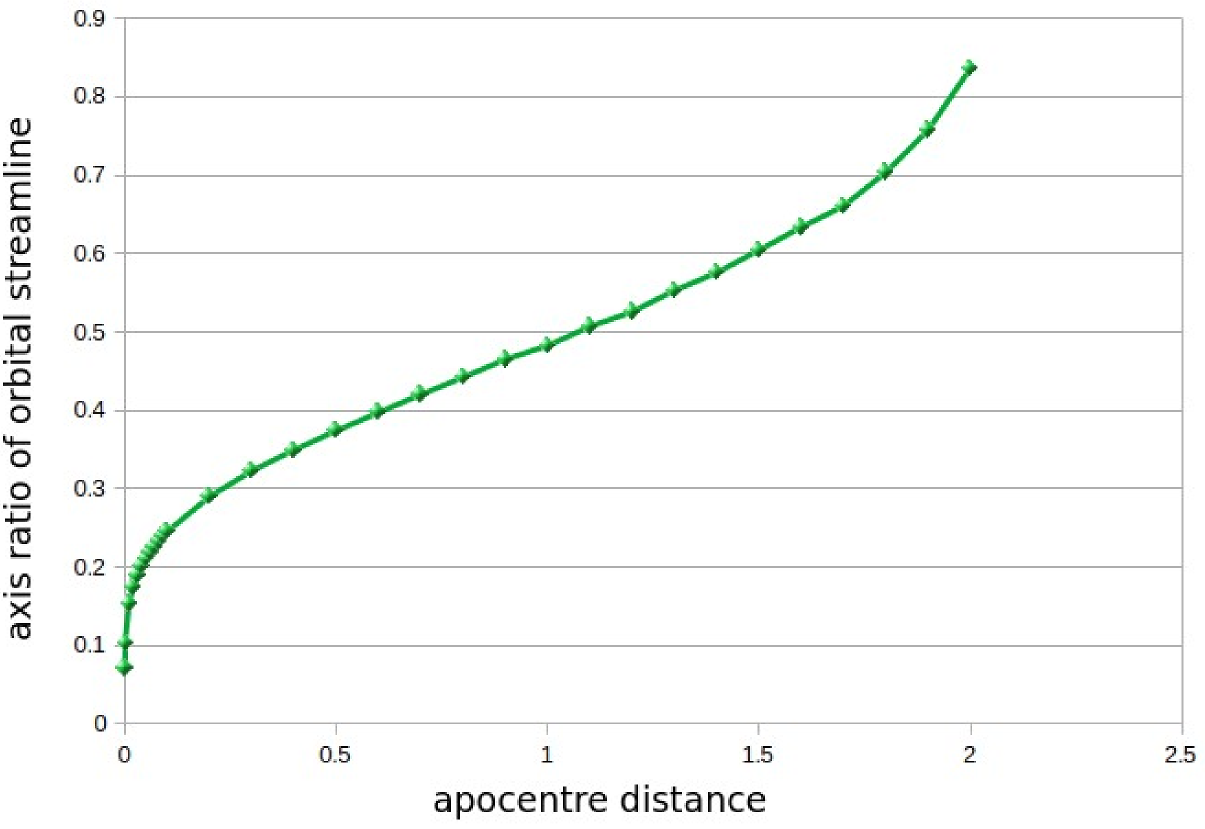

Figure 2: tuned axis ratios of orbital streamlines in field f ∝ r^0.75

The tuning of the streamlines succeeds up to an axis ratio of 0.8363 (see the largest streamline in fig 1). At axis ratios larger than that, it becomes increasingly difficult or impossible to achieve the reference precession rate. Therefore a maximum axis ratio of approximately 0.84 appears to define a limit beyond which this disk probably cannot be further extended outward.

The smallest streamline shown in Figure 1 has axis ratio of 0.1531. It is 200 times smaller than the largest streamline, but has an equal precession rate,. This is achieved because of its very different axis ratio (0.1531 versus 0.8363). It is possible to extend the disk even further inwards by adding even smaller streamlines.

The green data in Figure 3 shows the nearly equal precession rates which are achieved after the streamlines have been tuned by adjusting their axis ratios. In contrast the red data shows the diverse precession rates which occurred when every streamline had the same axis ratio = 0.4826.

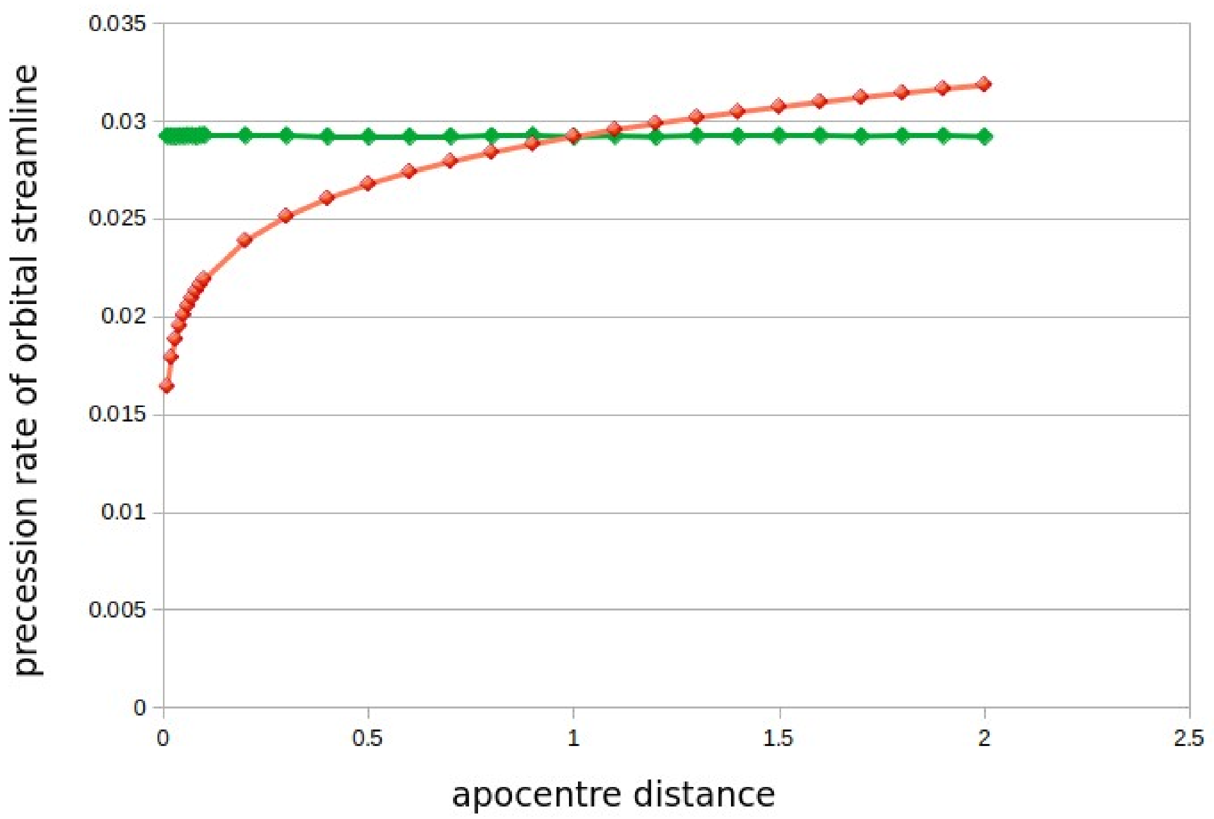

Figure 3: tuned (green) and untuned (red) precession rates in field f ∝ r^0.75

Data is listed in Figure 4. The apocentre velocities have been provided so that these findings can be replicated.

The precession rate of the streamlines is about 0.02921 radians per time unit, therefore the precession period is about 215 time units (this is the time it takes the shapes of the streamlines to precess around a complete circle).

Comparing the precession period with the orbital periods illustrates that during the time that it takes for the shapes of the streamlines to precess all the way around a complete circle, every particle in the largest streamline completes about 31.8 orbits, every particle in the reference streamline completes about 35.6 orbits, and every particle in the smallest streamline completre about 63.4 orbits. The precession is prograde (in the same direction as the orbits).

apocentre distance	apocentre speed	orbital period	axis ratio	precession rate (radians per unit time)
0.01	0.0029	3.3932	0.1531	0.02925
0.02	0.0060	3.7020	0.1745	0.02923
0.03	0.0093	3.8863	0.1889	0.02922
0.04	0.0127	4.0298	0.2005	0.02925
0.05	0.0162	4.1449	0.2101	0.02925
0.06	0.0197	4.2416	0.2186	0.02926
0.07	0.0233	4.3115	0.2261	0.02926
0.08	0.0269	4.3849	0.2323	0.02923
0.09	0.0307	4.4510	0.2393	0.02928
0.10	0.0345	4.5109	0.2453	0.02929
0.20	0.0745	4.9274	0.2898	0.02927
0.30	0.1180	5.1704	0.3227	0.02927
0.40	0.1640	5.3666	0.3494	0.02922
0.50	0.2130	5.5253	0.3740	0.02921
0.60	0.2650	5.6596	0.3974	0.02922
0.70	0.3200	5.7509	0.4200	0.02923
0.80	0.3780	5.8815	0.4422	0.02925
0.90	0.4400	5.9518	0.4651	0.02928
1.00	0.5000	6.0379	0.4826	0.02921
1.10	0.5700	6.1205	0.5070	0.02925
1.20	0.6370	6.1956	0.5258	0.02921
1.30	0.7170	6.2701	0.5529	0.02926
1.40	0.7950	6.3389	0.5755	0.02924
1.50	0.8850	6.4075	0.6044	0.02927
1.60	0.9800	6.4737	0.6339	0.02927
1.70	1.0750	6.5363	0.6606	0.02923
1.80	1.2000	6.6050	0.7036	0.02925
1.90	1.3500	6.6782	0.7578	0.02925
2.00	1.5500	6.7631	0.8363	0.02921

Figure 4: data for orbital streamlines in field $f \propto r$^0.75

References:

[1] Puel F, 1983, The Rotation Number of Bounded Orbits in a Central Field, 1983, 1983CeMec..29..255P

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[3] Valluri S R, Wilson C, Harper W, 1997, Newton's Apsidal Precession Theorem and Eccentric Orbits, 1997JHA....28...13V

[4] Valluri S R, Yu P, Smith G E, Wiegert P A, An extension of Newton's apsidal precession theorem, DOI:10.1111/j.1365-2966.2005.08819.x